Computer Science

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Systems Modelling and Analysis

Choose yourself and new technologies

L.17. Modeling of complex of operation systems

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Identification of complexes of operations with restricted measurements possibilities

- Description of complexes of operations.
- Identification of complexes of operations.
  - unlimited measurement possibilities,
  - limited possibilities of measurement of operations execution time,
  - limited possibilities of measurement of operations execution time and size of task or amount of resources allocated to an operation.
- Final remarks
- References

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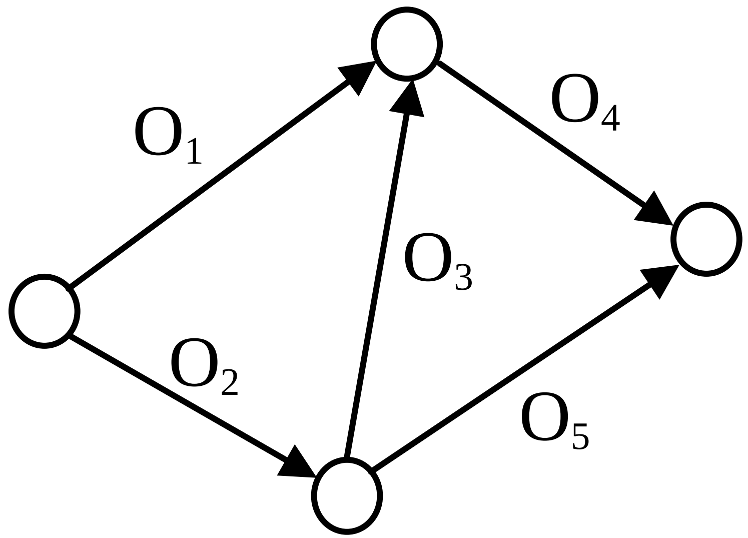
Description of complex of operations



Description of complex of operations

$O_1, O_2, \ldots, O_M$ – elementary static operations

For m-th operation description is given:

$$T_m = F_m(u_m, a_m), \quad m = 1,2,\ldots,M,$$

where $T_m \geq 0$ m-th operation completion time,

u_m – s_m-dimensional vector of m-th operation's inputs: $u_m \in U_m \subseteq \mathscr{R}^{+s_m}$,

a_m – r_m-dimensional vector of parameters: $a_m \in A_m \subseteq \mathscr{R}^{r_m}$,

F_m – known function: $F_m : U_m \times A_m \to \mathscr{R}^+$.

Description of complex of operations

Coordinates of the vector u_m stand for amount of resources or size of task for m-th operation.

Resources:

F_m – is nonincreasing function with respect to all of the vector u_m coordinates

For each a_m we have:

$$F_m(0_m, a_m) = \infty.$$

Tasks:

F_m – is nondecreasing function with respect to all of the vector u_m coordinates

For each a_m we have:

$$F_m(0_m, a_m) = 0.$$

Description of complex of operations

Structure of the system is described by the following graph:

$$G \subset \{1,2,\ldots,M\} \times \{1,2,\ldots,M\}$$

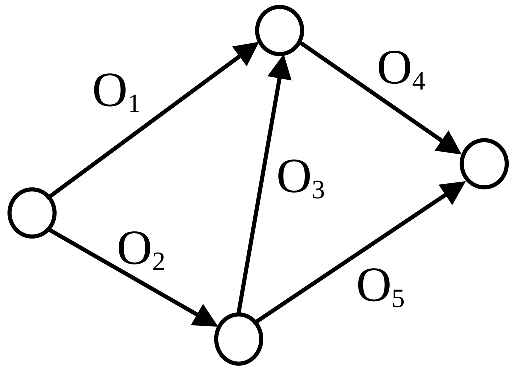
If $(m,n) \in G$ then the m-th operation is performed just after the n-th operation ends up.

The whole system **completion time**: $T = H(T_1, T_2, \ldots, T_M)$,

where H – function determining the complex of operations completion time, dependent on the complex of operations structure.

$$T = H(F_1(u_1, a_1), F_2(u_2, a_2), \ldots, F_M(u_M, a_M)) = F(u_1, u_2, \ldots, u_M, a_1, a_2, \ldots, a_M)$$

Identification of complex of operations



$$T_m = F_m(u_m, a_m), \quad m = 1,2,\ldots,M, \qquad T = H(T_1, T_2, \ldots, T_M)$$

H – function determining the total runtime of complex of operation

$F_1, F_2, \ldots, F_M$ – known functions

$a_1, a_2, \ldots, a_M$ – unknown parameters

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Unlimited measurement possibilities

Available measurements: $T_m(n), u_m(n), \quad m = 1,2,\ldots,M,$

where: $T_m(n)$ – measurement of m-th operation completion time for resource $u_m(n)$

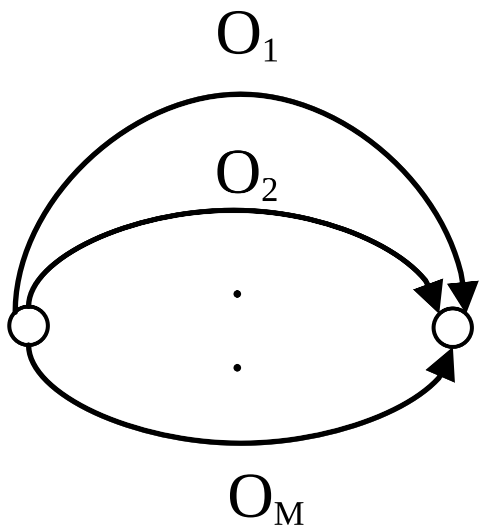
For each operation:

$$T_m(n) = F_m\big(u_m(n), a_m\big), \quad n = 1,2,\ldots,N.$$

Solving this system of equations with respect a_m results in identification algorithm for m-th operation,

Unlimited measurement possibilities

- Example – tasks allocation for complex of parallel operations



Complex of parallel operations

Unlimited measurement possibilities

- Example – tasks allocation for complex of parallel operations

For the complex of operations we have description:

$$T_m = a_m u_m, \quad a_m > 0,\ u_m \geq 0, \quad m = 1,2,\ldots,M$$

The total size of all taks is u.

Solution of tasks allocation problem should satisfy the following constraints:

$$\mathscr{D}_u = \left\{ u_m \geq 0,\ m = 1,2,\ldots,M;\ \sum_{m=1}^{M} u_m = u \right\}$$

Unlimited measurement possibilities

- Example – tasks allocation for complex of parallel operations

For each operation we have: $T_m(n) = a_m u_m(n)$

Note, that for m-th operation one measurement is enough.

Parameter of m-th operation's description we evaluate as:

$$a_m = \frac{T_m(n)}{u_m(n)}, \quad m = 1,2,\ldots,M$$

Limited possibilities of measurement of operations execution time

Available measurements: $T(n), u_1(n), u_2(n), \ldots, u_M(n), \quad n = 1,2,\ldots,N,$

where: $T(n)$ – measurement of m-th operation completion time for resource $u_m(n)$

N – number of experiment's repetitions $m = 1,2,\ldots,M$

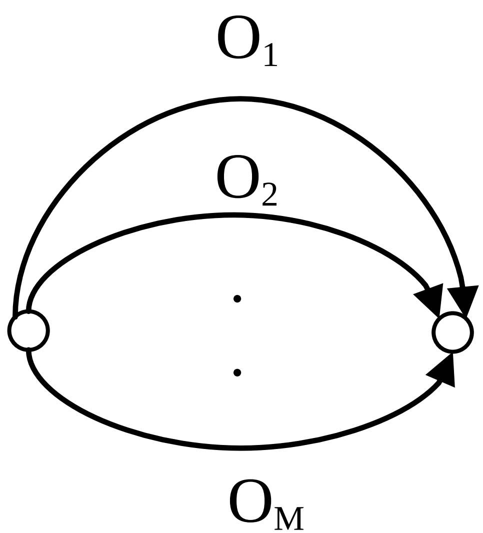
Observed completion time of complex of operations for measured data is given by:

$$T(n) = F\left(u_1(n), u_2(n), \ldots, u_M(n), a_1, a_2, \ldots, a_M\right), \quad n = 1,2,\ldots,N$$

Solving this system of equations with respect $a_1, a_2, \ldots, a_M$ results in identification algorithm.

Limited possibilities of measurement of operations execution time

- Example – tasks allocation for complex of parallel operations



Complex of parallel operations

Limited possibilities of measurement of operations execution time

- Example – tasks allocation for complex of parallel operations

Total completion time for the whole complex of operations is given by:

$$T = \max_{1 \le m \le M} \{a_m u_m\}$$

For measurement data we have:

$$T(n) = \max_{1 \le m \le M} \{a_m u_m(n)\}, \quad n = 1, 2, \ldots, N.$$

Solving this system of equations with respect $a_1, a_2, \ldots, a_M$ results in identification algorithm.

Limited possibilities of measurement of operations execution time

- Example – tasks allocation for complex of parallel operations

Total completion time for the whole complex of operations is given by:

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Solving this system of equations with respect $a_1, a_2, \ldots, a_M$ results in identification algorithm.

Limited possibilities of measurement of operations execution time

- Example – tasks allocation for complex of parallel operations

For n-th run of complex of operations we allocate all resources or tasks to a single operation:

$$u_m(m) = u(m), \quad u_m(n) = 0, \quad n = 1, 2, \ldots, M, \; n \neq m$$

For such an experiment we have:

$$T(m) = a_m u_m(m), \quad m = 1, 2, \ldots, M.$$

Solving this system of equations with respect $a_1, a_2, \ldots, a_M$ results in identification algorithm in the form:

$$a_m = \frac{T(m)}{u_m(m)}, \quad m = 1, 2, \ldots, M.$$

Limited possibilities of measurement of operations execution time

- Example – tasks allocation for complex of parallel operations

We allocate all resources or tasks uniformly to each operation:

$$u_1(n) = u_2(n) = \cdots = u_M(n) = \bar{u}(n) = \frac{u(n)}{M}.$$

For such an experiment we have:

$$T(n) = \bar{u}(n) \max_{1 \le m \le M} \{a_m\}, \quad n = 1, 2, \ldots, N.$$

Solution of the system of equation is not unique with respect $a_1, a_2, \ldots, a_M$.
We are only able to work out a parameter, which is a function of parameters $a_1, a_2, \ldots, a_M$:

$$\max_{1 \le m \le M} \{a_m\} = \frac{T(n)}{\bar{u}(n)}.$$

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Limited possibilities of measurement of operations execution time and size of task or amount of resources allocated to an operation

Available measurements: $T^*(n), u(n), \quad n = 1, 2, \ldots, N,$

where: $T^*(n)$ – optimal completion time for the maximal size of task or the global resource $u_m(n)$

Following assumption about optimal task or resources allocation, we take the following allocation into consideration:

$$u_1^*, u_2^*, \ldots, u_M^*.$$

For such allocation the completion time is minimal:

$$F\left(u_1^*, u_2^*, \ldots, u_M^*, a_1, a_2, \ldots, a_M\right) = \min_{(u_1, u_2, \ldots, u_M) \in \mathscr{D}_u} F\left(u_1, u_2, \ldots, u_M, a_1, a_2, \ldots, a_M\right).$$

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Limited possibilities of measurement of operations execution time and size of task or amount of resources allocated to an operation

Solution of the problem results in optimal algorithms of allocation:

$$u_m^* = G_m(u, a_1, a_2, \ldots, a_M), \quad m = 1,2,\ldots,M.$$

Optimal completion time for complex of operations:

$$T^* = F(u_1^*, u_2^*, \ldots, u_M^*, a_1, a_2, \ldots, a_M) =$$
$$= F(G_1(u, a_1, a_2, \ldots, a_M), \ldots, G_M(u, a_1, a_2, \ldots, a_M), a_1, a_2, \ldots, a_M) = \widetilde{F}(u, a_1, a_2, \ldots, a_M)$$

For observed measurement data we may propose the following system of equations:

$$T^*(n) = \widetilde{F}(u(n), a_1, a_2, \ldots, a_M), \quad n = 1,2,\ldots,N.$$

Solving this system of equations with respect $a_1, a_2, \ldots, a_M$ results in identification algorithm.

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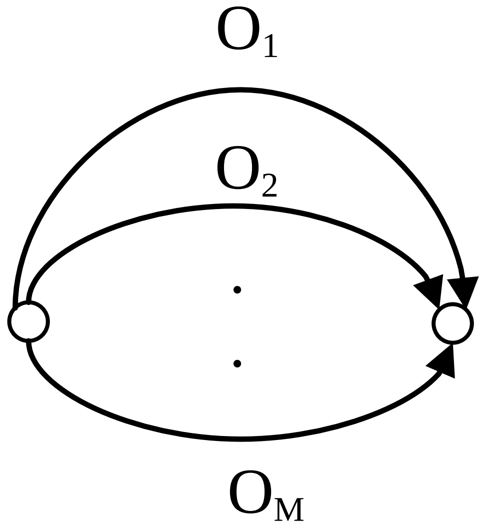
Limited possibilities of measurement of operations execution time and size of task or amount of resources allocated to an operation

- Example – tasks allocation for complex of parallel operations



Limited possibilities of measurement of operations execution time and size of task or amount of resources allocated to an operation

- Example – tasks allocation for complex of parallel operations

The completion time of complex of operations is optimal as long as all operations are completed at the same moment:

$$T^* = T_1^* = T_2^* = \ldots = T_M^*.$$

Taking description of operations and constraints, optimal task allocation satisfies the following system of equations:

$$T^* = a_m u_m, \quad m = 1,2,\ldots,M, \quad \sum_{m=1}^{M} u_m = u.$$

Limited possibilities of measurement of operations execution time and size of task or amount of resources allocated to an operation

- Example – tasks allocation for complex of parallel operations

Solving the system of equations with respect $u_1, u_2, \ldots, u_M$ results in **optimal allocation algorithm** in the form:

$$u_m^* = \frac{u}{a_m \sum_{m=1}^{M} \frac{1}{a_m}}, \quad m = 1,2,\ldots,M.$$

Optimal completion time is expressed by the formula: $T^* = \dfrac{u}{\sum_{m=1}^{M} \frac{1}{a_m}}.$

Limited possibilities of measurement of operations execution time and size of task or amount of resources allocated to an operation

- Example – tasks allocation for complex of parallel operations

For measurement data we have: $$T^*(n) = \frac{u(n)}{\sum_{m=1}^{M} \frac{1}{a_m}}, \quad n = 1,2,\ldots,N.$$

Solution of the system of equation is not unique with respect $a_1, a_2, \ldots, a_M$.
We are only able to work out a parameter, which is a function of parameters $a_1, a_2, \ldots, a_M$:

$$\frac{u(n)}{\sum_{m=1}^{M} \frac{1}{a_m}} = \frac{T^*}{u(n)}, \quad n = 1,2,\ldots,N.$$

Obtained value may be treated as parameter of operation equivalent to the whole complex of operations.

Final remarks

- Identification of complexes of operations.
  - unlimited measurement possibilities,
  - limited possibilities of measurement of operations execution time,
  - limited possibilities of measurement of operations execution time and size of task or amount of resources allocated to an operation.
- The problem of separability.

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Thank you for attention