Computer Science

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Systems Modelling and Analysis

Choose yourself and new technologies

L.13. Identification of dynamic plants

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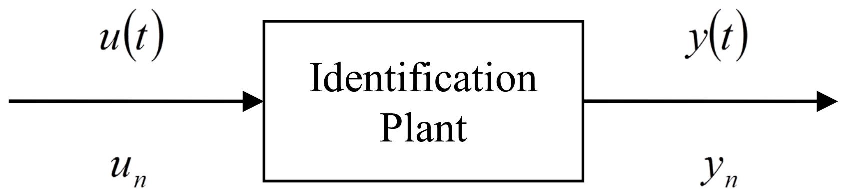
Identification of Dynamic Plants



Model equation: $y_n + a_1 y_{n-1} + \ldots + a_m y_{n-m} = b_0 u_n + b_1 u_{n-1} + \ldots + b_k u_{n-k}$

or using transfer function:

$$A\left(z^{-1}\right) = a_1 z^{-1} + a_2 z^{-2} + \ldots + a_m z^{-m}$$
$$B\left(z^{-1}\right) = b_0 + b_1 z^{-1} + \ldots + b_k z^{-k}$$

we can write equivalently:

$$y_n + A\left(z^{-1}\right) y_n = B\left(z^{-1}\right) u_n$$
$$y_n \left(1 + A\left(z^{-1}\right)\right) = B\left(z^{-1}\right) u_n$$

$$K\left(z^{-1}\right) = \frac{B\left(z^{-1}\right)}{\left(1 + K\left(z^{-1}\right)\right)}$$

Identification of Dynamic Plants

Let us denote:

$$\theta = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \\ b_0 \\ b_1 \\ \vdots \\ b_k \end{bmatrix} \qquad \mathrm{X}_n = \begin{bmatrix} -y_{n-1} \\ -y_{n-2} \\ \vdots \\ -y_{n-m} \\ u_n \\ u_{n-1} \\ \vdots \\ u_{n-k} \end{bmatrix}$$

and rewrite model equation as:

$$y_n = -a_1 y_{n-1} - \ldots - a_m y_{n-m} + b_0 u_n + b_1 u_{n-1} + \cdots + b_k u_{n-k}$$

$$y_n = \theta^T \mathrm{X}_n$$

Identification of Dynamic Plants
Least Squares method

Performance index:

$$Q_N(\theta) = \sum_{n=0}^{N} \left(y_n - \theta^T \mathbf{X}_n\right)^2$$

Optimization problem:

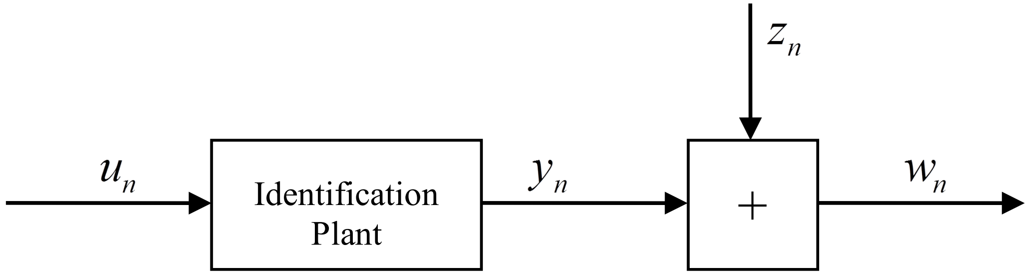
$$\theta_N \to Q_N(\theta_N) = \min_{\theta \in \Theta} Q_N(\theta) = \min_{\theta \in \Theta} \sum_{n=0}^{N} \left(y_n - \theta^T \mathbf{X}_n\right)^2$$

$$Q_N(\theta) = \sum_{n=0}^{N} \left(y_n - \theta^T \mathbf{X}_n\right)^2$$

Solution:

$$\theta_N = \left(\sum_{n=0}^{N} \mathbf{X}_n \mathbf{X}_n^T \right)^{-1} \sum_{n=0}^{N} y_n X_n$$

Identification of Dynamic Plants
Estimation of Linear Plant Parameters



$$y_n + a_1 y_{n-1} + \ldots + a_m y_{n-m} = b_0 u_n + b_1 u_{n-1} + \cdots + b_k u_{n-k}$$

Measurements with disturbances:

$$w_n = y_n + z_n \qquad y_n = w_n - z_n$$

$$w_n + a_1 w_{n-1} + \ldots + a_m w_{n-m} = b_0 u_n + b_1 u_{n-1} + \ldots + b_k u_{n-k} + \overbrace{z_n + a_1 z_{n-1} + \ldots + a_m z_{n-m}}^{v_n}$$

z is now correlated with previous states (we had $w = F(u,\theta) + z$ before)

Identification of Dynamic Plants
Estimation of Linear Plant Parameters

or by transfer function

$$A\left(z^{-1}\right)=a_1 z^{-1}+a_2 z^{-2}+\ldots+a_m z^{-m}$$
$$B\left(z^{-1}\right)=b_0+b_1 z^{-1}+\ldots+b_k z^{-k}$$

we can rewrite equivalently:

$$w_n=A\left(z^{-1}\right)w_n+B\left(z^{-1}\right)u_n+v_n$$
$$w_n\left(1+A\left(z^{-1}\right)\right)=B\left(z^{-1}\right)u_n$$

$$w_n=-a_1 w_{n-1}-a_2 w_{n-2}-\ldots-a_m w_{n-m}+b_0 u_n+b_1 u_{n-1}+\ldots+b_k u_{n-k}+v_n$$

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Identification of Dynamic Plants

Estimation of Linear Plant Parameters

Let us denote:

$$\theta = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \\ b_0 \\ b_1 \\ \vdots \\ b_k \end{bmatrix} \qquad \mathbf{X}_n = \begin{bmatrix} -w_{n-1} \\ -w_{n-2} \\ \vdots \\ -w_{n-m} \\ u_n \\ u_{n-1} \\ \vdots \\ u_{n-k} \end{bmatrix}$$

and rewrite model equation as:

$$w_n = \theta^T \mathrm{X}_n + v_n$$

$$v_n = z_n + a_1 z_{n-1} + \ldots + a_m z_{n-m}$$

Identification of Dynamic Plants
Estimation of Linear Plant Parameters

1. Direct approach

$$w_n = -A(z^{-1})w_n + B(z^{-1})u_n + v_n = -A(z^{-1})w_n + B(z^{-1})u_n + (1 + A(z^{-1}))z_n$$

$$w_n(1 - A(z^{-1})) = B(z^{-1})u_n + (1 + A(z^{-1}))z_n$$

$$\frac{w_n(1 + A(z^{-1}))}{1 + A(z^{-1})} = w_n = \frac{B(z^{-1})u_n}{1 + A(z^{-1})} + z_n$$

z is de-correlated, but parameters are nonlinearly involved in the equation.

Identification of Dynamic Plants

Estimation of Linear Plant Parameters

LSQ method, forgetting that are correlated:

Performance index:

$$Q_N(\theta) = \sum_{n=0}^{N} \left(w_n - \theta^T X_n\right)^2$$

$$\nabla_\theta Q_N(\theta) = -2\sum_{n=0}^{N} \left(w_n - \theta^T X_n\right) X_n = 0_{m+k+1}$$

Solution:

$$\theta_N = \left(\sum_{n=0}^{N} X_n X_n^T\right)^{-1} \sum_{n=0}^{N} w_n X_n$$

But: v_n and z_n are correlated

$$v_n = z_n + a_1 z_{n-1} + \ldots + a_m z_{n-m}$$

Identification of Dynamic Plants

Estimation of Linear Plant Parameters

2. De-correlation of disturbances

Let us assume, that $v_n = \rho v_{n-1} + z_n$

$$v_n\left(1-\rho z^{-1}\right) = z_n$$

By multiplying both sides of equation:

$$w_n + a_1 w_{n-1} + \ldots + a_m w_{n-m} = b_0 u_n + b_1 u_{n-1} + \ldots + b_k u_{n-k} + \overbrace{z_n + a_1 z_{n-1} + \ldots + a_m z_{n-m}}^{v_n}$$

by $\left(1-\rho z^{-1}\right)$, we obtain:

$$w_n\left(1-\rho z^{-1}\right) = -a_1 w_{n-1}\left(1-\rho z^{-1}\right) - \ldots - a_m w_{n-m}\left(1-\rho z^{-1}\right) +$$
$$+ b_0 u_n\left(1-\rho z^{-1}\right) + \ldots + b_k u_{n-k}\left(1-\rho z^{-1}\right) + v_n\left(1-\rho z^{-1}\right)$$

Identification of Dynamic Plants
Estimation of Linear Plant Parameters

Let $w_n^f = \left(1 - \rho z^{-1}\right) w_n$, $u_n^f = \left(1 - \rho z^{-1}\right) u_n$, where f stands for *filtered* . Then:

$$w_n^f = -a_1 w_{n-1}^f - \ldots - a_m w_{n-m}^f + b_0 u_n^f + b_1 u_{n-1}^f + \ldots + b_k u_{n-k}^f + z_n$$

and now we have grounds to apply LS method:

$$\theta_N = \left(\sum_{n=0}^{N} \mathbf{X}_n^f \left(\mathbf{X}_n^f\right)^T \right)^{-1} \sum_{n=0}^{N} w_n^f X_n^f$$

where u_n^f and w_n^f are work out iteratively with use of following procedure:

Identification of Dynamic Plants

Estimation of Linear Plant Parameters

step 1.

$$\theta_N = \left(\sum_{n=0}^{N} \mathbf{X}_n \mathbf{X}_n^T \right)^{-1} \sum_{n=0}^{N} w_n X_n$$

step 2.

$$w_n = \theta_N^T \mathrm{X}_n + v_n$$

$$v_n = w_n - \theta_N^T \mathrm{X}_n$$

step 3.

$$v_n = v_{n-1}\rho + z_n$$

$$\min_{\rho} \sum_{n=1}^{N} (v_n - \rho v_{n-1})^2 \Rightarrow \rho_N = \frac{\sum_{n=1}^{N} v_n v_{n-1}}{\sum_{n=1}^{N} v_{n-1}^2}$$

Identification of Dynamic Plants
Estimation of Linear Plant Parameters

step 4.

$$w_n^f = \left(1 - \rho_N z^{-1}\right) w_n = w_n - \rho_N w_{n-1}$$

$$u_n^f = \left(1 - \rho_N z^{-1}\right) u_n = u_n - \rho_N u_{n-1}$$

step 5.

$$\theta_N^f = \left(\sum_{n=0}^{N} \mathbf{X}_n^f \left(\mathbf{X}_n^f\right)^T \right)^{-1} \sum_{n=0}^{N} w_n^f X_n^f$$

GOTO 2

Identification of Dynamic Plants
Estimation of Linear Plant Parameters

3. Generalized Least Square Method

$$D(z^{-1}) = (d_1 z^{-1} + d_2 z^{-2} \cdots d_r z^{-r})$$

It means that we take into account correlation (back in time) between disturbances, not only ρz^{-1}, as before.

$$v_n = D(z^{-1}) v_n + z_n$$

$$v_n = d_1 v_{n-1} + d_2 v_{n-2} + \ldots + d_r v_{n-r} + z_n$$

$$v_n (1 - D(z^{-1})) = z_n$$

Identification of Dynamic Plants

Estimation of Linear Plant Parameters

3. Generalized Least Square Method

By multiplying both sides of equation:

$$w_n + a_1 w_{n-1} + \ldots + a_m w_{n-m} = b_0 u_n + b_1 u_{n-1} + \ldots + b_k u_{n-k} + \overbrace{z_n + a_1 z_{n-1} + \ldots + a_m z_{n-m}}^{v_n}$$

by $\left(1 - D\left(z^{-1}\right)\right)$, we obtain:

$$w_n^f = w_n\left(1 - D\left(z^{-1}\right)\right) \qquad u_n^f = u_n\left(1 - D\left(z^{-1}\right)\right)$$

$$w_n^f = -a_1 w_{n-1}^f - \ldots - a_m w_{n-m}^f + b_0 u_n^f + \ldots + b_m u_{n-m}^f + z_n$$

and iterative procedure is performed:

Identification of Dynamic Plants

Estimation of Linear Plant Parameters

3. Generalized Least Square Method

step 1.

$$\theta_N = \left(\sum_{n=0}^{N} \mathbf{X}_n \mathbf{X}_n^T \right)^{-1} \sum_{n=0}^{N} w_n X_n$$

step 2.

$$v_n = w_n - \theta_N^T \mathrm{X}_n$$

step 3.

$$v_n = d_1 v_{n-1} + d_2 v_{n-2} + \ldots + d_r v_{n-r} + z_n$$

Identification of Dynamic Plants

Estimation of Linear Plant Parameters

3. Generalized Least Square Method

By denoting $\Omega = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_r \end{bmatrix}, \overline{v}_n = \begin{bmatrix} v_{n-1} \\ v_{n-2} \\ \vdots \\ v_{n-r} \end{bmatrix}$ we can write equivalently:

$$v_n = \Omega^T \overline{v}_n + z_n$$

$$\Omega_N \to \min_{\Omega} \sum_{n=r}^{N} \left(v_n - \Omega^T \overline{v}_n \right)^2 \Rightarrow \Omega_N = \left(\sum_{n=r}^{N} \overline{v}_n \overline{v}_n^T \right)^{-1} \sum_{n=r}^{N} v_n \overline{v}_n$$

which gives us values of parameters of polynomial D.

Identification of Dynamic Plants

Estimation of Linear Plant Parameters

3. Generalized Least Square Method

step 4.

$$w_n^f = w_n\left(1 - D\left(z^{-1}\right)\right) \qquad \leftarrow \quad \Omega_N$$

$$u_n^f = u_n\left(1 - D\left(z^{-1}\right)\right) \qquad \leftarrow \quad \Omega_N$$

step 5.

$$\theta_N^f = \left(\sum_{n=0}^{N} \mathbf{X}_n^f \left(\mathbf{X}_n^f\right)^T\right)^{-1} \sum_{n=0}^{N} w_n^f X_n^f$$

GOTO 2

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Identification of Dynamic Plants

Recursive Algorithms

$$\theta_N = \left(\sum_{n=1}^{N} \mathrm{x}_n \mathrm{x}_n^T \right)^{-1} \sum_{n=1}^{N} \mathrm{x}_n w_n$$

a new measurement $\mathbf{x}_{N+1}, \ w_{N+1}$ comes in.

Update algorithm:

$$\theta_{N+1} = \Psi_R\left(\theta_N, \mathrm{x}_{N+1}, w_{N+1}\right)$$

$$\theta_{N+1} = \left(\sum_{n=1}^{N+1} \mathrm{x}_n \mathrm{x}_n^T \right)^{-1} \sum_{n=1}^{N+1} \mathrm{x}_n w_n = \left(\sum_{n=1}^{N} \mathrm{x}_n \mathrm{x}_n^T + \mathrm{x}_{N+1} \mathrm{x}_{N+1}^T \right)^{-1} \left(\sum_{n=1}^{N} \mathrm{x}_n w_n + \mathrm{x}_{N+1} w_{N+1} \right)$$

Identification of Dynamic Plants

Useful Transformations:

$$\left(\mathbf{A}+\mathbf{B}\mathbf{D}^{-1}\mathbf{C}\right)^{-1}=\mathbf{A}^{-1}-\mathbf{A}^{-1}\mathbf{B}\left(\mathbf{D}+\mathbf{C}\mathbf{A}^{-1}\mathbf{B}\right)^{-1}\mathbf{C}\mathbf{A}^{-1}$$

$\mathbf{B}=\mathbf{x}$ – column vector

$\mathbf{D}^{-1}=1$

$\mathbf{C}=\mathbf{x}^{T}$

$$\left(\mathbf{A}+\mathbf{x}\mathbf{x}^{T}\right)^{-1}=\mathbf{A}^{-1}-\mathbf{A}^{-1}\mathbf{x}\left(1+\mathbf{x}^{T}\mathbf{A}^{-1}\mathbf{x}\right)^{-1}\mathbf{x}^{T}\mathbf{A}^{-1}$$

$\mathbf{D}^{-1}=-1$

$$\left(\mathbf{A}-\mathbf{x}\mathbf{x}^{T}\right)^{-1}=\mathbf{A}^{-1}+\mathbf{A}^{-1}\mathbf{x}\left(1-\mathbf{x}^{T}\mathbf{A}^{-1}\mathbf{x}\right)^{-1}\mathbf{x}^{T}\mathbf{A}^{-1}$$

$\mathbf{D}^{-1}=\alpha$

$$\left(\mathbf{A}+\alpha\mathbf{x}\mathbf{x}^{T}\right)^{-1}=\mathbf{A}^{-1}-\mathbf{A}^{-1}\mathbf{x}\left(\frac{1}{\alpha}+\mathbf{x}^{T}\mathbf{A}^{-1}\mathbf{x}\right)^{-1}\mathbf{x}^{T}\mathbf{A}^{-1}$$

Identification of Dynamic Plants

Recursive Algorithms

$$\left(\sum_{n=1}^{N+1}\mathbf{x}_n\mathbf{x}_n^T\right)^{-1}=\left(\sum_{n=1}^{N}\mathbf{x}_n\mathbf{x}_n^T+\mathbf{x}_{N+1}\mathbf{x}_{N+1}^T\right)^{-1}=$$

$$=\left(\sum_{n=1}^{N}\mathbf{x}_n\mathbf{x}_n^T\right)^{-1}-\left(\sum_{n=1}^{N}\mathbf{x}_n\mathbf{x}_n^T\right)^{-1}\frac{\mathbf{x}_{N+1}\mathbf{x}_{N+1}^T}{1+\mathbf{x}_{N+1}^T\left(\sum_{n=1}^{N}\mathbf{x}_n\mathbf{x}_n^T\right)^{-1}\mathbf{x}_{N+1}}\left(\sum_{n=1}^{N}\mathbf{x}_n\mathbf{x}_n^T\right)^{-1}$$

Let $P_N=\left(\sum_{n=1}^{N}\mathbf{x}_n\mathbf{x}_n^T\right)^{-1}$

$$P_{N+1}=P_N-P_N\frac{\mathbf{x}_{N+1}\mathbf{x}_{N+1}^T}{1+\mathbf{x}_{N+1}^T P_N\mathbf{x}_{N+1}}P_N$$

We can rewrite previous equation as:

Identification of Dynamic Plants

Recursive Algorithms

$$\theta_{N+1} = \left(P_N - P_N \frac{\mathbf{x}_{N+1}\mathbf{x}_{N+1}^T}{1+\mathbf{x}_{N+1}^T P_N \mathbf{x}_{N+1}} P_N \right)\left(\sum_{n=1}^{N} \mathbf{x}_n w_n + \mathbf{x}_{N+1} w_{N+1} \right) =$$

$$= P_N \sum_{n=1}^{N} \mathbf{x}_n w_n + P_N \mathbf{x}_{N+1} w_{N+1} - \frac{P_N \mathbf{x}_{N+1}\mathbf{x}_{N+1}^T P_N}{1+\mathbf{x}_{N+1}^T P_N \mathbf{x}_{N+1}} \sum_{n=1}^{N} \mathbf{x}_n w_n - \frac{P_N \mathbf{x}_{N+1}\mathbf{x}_{N+1}^T P_N \mathbf{x}_{N+1} w_{N+1}}{1+\mathbf{x}_{N+1}^T P_N \mathbf{x}_{N+1}} =$$

$$= P_N \sum_{n=1}^{N} \mathbf{x}_n w_n + \frac{P_N \mathbf{x}_{N+1} w_{N+1} + P_N \mathbf{x}_{N+1} w_{N+1} \mathbf{x}_{N+1}^T P_N \mathbf{x}_{N+1} - P_N \mathbf{x}_{N+1} w_{N+1} \mathbf{x}_{N+1}^T P_N \mathbf{x}_{N+1} - P_N \mathbf{x}_{N+1}\mathbf{x}_{N+1}^T P_N \sum_{n=1}^{N} \mathbf{x}_n w_n}{1+\mathbf{x}_{N+1}^T P_N \mathbf{x}_{N+1}} =$$

$$= P_N \sum_{n=1}^{N} \mathbf{x}_n w_n + \frac{P_N \mathbf{x}_{N+1} w_{N+1} - P_N \mathbf{x}_{N+1}\mathbf{x}_{N+1}^T P_N \sum_{n=1}^{N} \mathbf{x}_n w_n}{1+\mathbf{x}_{N+1}^T P_N \mathbf{x}_{N+1}} =$$

$$= \theta_N + K_{N+1}\left(w_{N+1} - \mathbf{x}_{N+1}^T \theta_N \right)$$

where: $K_{N+1} = \dfrac{P_N \mathbf{x}_{N+1}}{1+\mathbf{x}_{N+1}^T P_N \mathbf{x}_{N+1}}$

Identification of Dynamic Plants
Recursive Algorithms

Taking everything together, we obtain the *on-line identification* algorithm

$$\theta_{N+1} = \theta_N + K_{N+1}\left(w_{N+1} - \mathbf{x}_{N+1}^T \theta_N\right)$$

$$K_{N+1} = \frac{P_N \mathbf{x}_{N+1}}{1 + \mathbf{x}_{N+1}^T P_N \mathbf{x}_{N+1}}$$

$$P_{N+1} = P_N - \frac{P_N \mathbf{x}_{N+1} \mathbf{x}_{N+1}^T P_N}{1 + \mathbf{x}_{N+1}^T P_N \mathbf{x}_{N+1}}$$

Identification of Dynamic Plants

Time-Variant Plants – Parameters Change With Time

1. Including previous measurements with weights

$$\theta_N \rightarrow \min \sum_{n=1}^{N} \beta_n \left(w_n - \theta^T \mathrm{x}_n\right)^2$$

How to choose values of β_n?

- When n is small β should be also small (close to zero).
- When n is large β should be also large (close to N).

Identification of Dynamic Plants

Time-Variant Plants – Parameters Change With Time

$$\beta_n = \alpha^{N-n}\ , \quad \alpha \in \left(\tfrac{1}{2}, 1\right)$$

Optimization problem:

$$\theta_N \rightarrow \min \sum_{n=1}^{N} \alpha^{N-n} \left(w_n - \theta^T \mathrm{x}_n \right)^2$$

Solution:

$$\theta_N = \left(\sum_{n=1}^{N} \alpha^{N-n} \mathrm{x}_n \mathrm{x}_n^T \right)^{-1} \sum_{n=1}^{N} \alpha^{N-n} \mathrm{x}_n w_n$$

$$\theta_{N+1} = \left(\sum_{n=1}^{N+1} \alpha^{N+1-n} \mathrm{x}_n \mathrm{x}_n^T \right)^{-1} \sum_{n=1}^{N+1} \alpha^{N+1-n} \mathrm{x}_n w_n =$$

$$= \left(\alpha \sum_{n=1}^{N} \alpha^{N-n} \mathrm{x}_n \mathrm{x}_n^T + \mathrm{x}_{N+1} \mathrm{x}_{N+1}^T \right)^{-1} \left(\alpha \sum_{n=1}^{N} \alpha^{N-n} \mathrm{x}_n w_n + \mathrm{x}_{N+1} w_{N+1} \right)$$

$$\vdots$$

Identification of Dynamic Plants

Time-Variant Plants – Parameters Change With Time

Taking everything together, we obtain the *identification* algorithm

$$\theta_{N+1} = \theta_N + K_{N+1}\left(w_{N+1} - \mathbf{x}_{N+1}^T \theta_N\right)$$

$$K_{N+1} = \frac{P_N \mathbf{x}_{N+1}}{\alpha + \mathbf{x}_{N+1}^T P_N \mathbf{x}_{N+1}}$$

$$P_{N+1} = \frac{1}{\alpha}\left(P_N - \frac{P_N \mathbf{x}_{N+1} \mathbf{x}_{N+1}^T P_N}{1 + \mathbf{x}_{N+1}^T P_N \mathbf{x}_{N+1}}\right)$$

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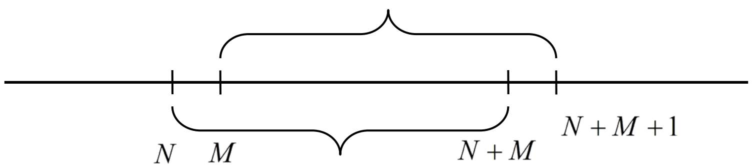
Identification of Dynamic Plants

Shifting window

We use only M last measurements for identification. After getting new measurement, the last one is thrown out.



$\theta_{N+M,M}$

$\theta_{N+M+1,M+1}$ add measurement $\mathbf{x}_{N+M+1}, w_{N+M+1}$

$\theta_{N+M+1,M}$ reject measurement $\mathbf{x}_N, w_N$

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Identification of Dynamic Plants

Shifting window

Optimization problem:

$$\theta_{N+M,M} \to \min_{\theta} \sum_{n=N}^{N+M} \left(w_n - \theta^T \mathrm{x}_n\right)^2$$

Identification algorithm:

$$\theta_{N+M,M} = \left(\sum_{n=N}^{N+M} \mathrm{x}_n \mathrm{x}_n^T \right)^{-1} \sum_{n=N}^{N+M} \mathrm{x}_n w_n$$

Identification of Dynamic Plants

Shifting window

step 1.

$$\theta_{N+M+1,M+1} = \left(\sum_{n=N}^{N+M+1} \mathrm{x}_n \mathrm{x}_n^T \right)^{-1} \sum_{n=N}^{N+M+1} \mathrm{x}_n w_n$$

(add $\mathbf{x}_{N+M+1}$, w_{N+M+1})

step 2.

$$\theta_{N+M+1,M} = \left(\sum_{n=N+1}^{N+M+1} \mathrm{x}_n \mathrm{x}_n^T \right)^{-1} \sum_{n=N+1}^{N+M+1} \mathrm{x}_n w_n$$

(reject $\mathbf{x}_N$, w_N)

Identification of Dynamic Plants

Shifting window

Let $P_{N+M,M} = \left(\sum_{n=N}^{N+M} \mathbf{x}_n \mathbf{x}_n^T \right)^{-1}$

step 1.

$$P_{N+M+1,M+1} = \left(\sum_{n=N}^{N+M+1} \mathbf{x}_n \mathbf{x}_n^T \right)^{-1}$$

$$\mathrm{u}_{N+M+1,M+1} = \theta_{N+M,M} + K_{N+M+1,M+1}\left(w_{N+M+1} - \mathrm{x}_{N+M+1}^T \theta_{N+M,M}\right)$$

$$K_{N+M+1,M+1} = \frac{P_{N+M,M}\mathrm{x}_{N+M+1}}{1 + \mathrm{x}_{N+M+1}^T P_{N+M,M}\mathrm{x}_{N+M+1}}$$

$$P_{N+M+1,M+1} = P_{N+M,M} - \frac{P_{N+M,M}\mathrm{x}_{N+M+1}\mathrm{x}_{N+M+1}^T P_{N+M,M}}{1 + \mathrm{x}_{N+M+1}^T P_{N+M,M}\mathrm{x}_{N+M+1}}$$

Identification of Dynamic Plants
Shifting window

step 2.

$$\theta_{N+M+1,M} = \theta_{N+M,M+1} - K_{N+M+1,M}\left(w_N - \mathbf{x}_N^T \theta_{N+M+1,M+1}\right)$$

$$K_{N+M+1,M} = \frac{P_{N+M+1,M+1}\mathbf{x}_N}{1 - \mathbf{x}_N^T P_{N+M+1,M+1}\mathbf{x}_N}$$

$$P_{N+M+1,M} = P_{N+M+1,M+1} + P_{N+M+1,M+1}\frac{\mathbf{x}_N \mathbf{x}_N^T}{1 - \mathbf{x}_N^T P_{N+M+1,M+1}\mathbf{x}_N}P_{N+M+1,M+1}$$

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Thank you for attention