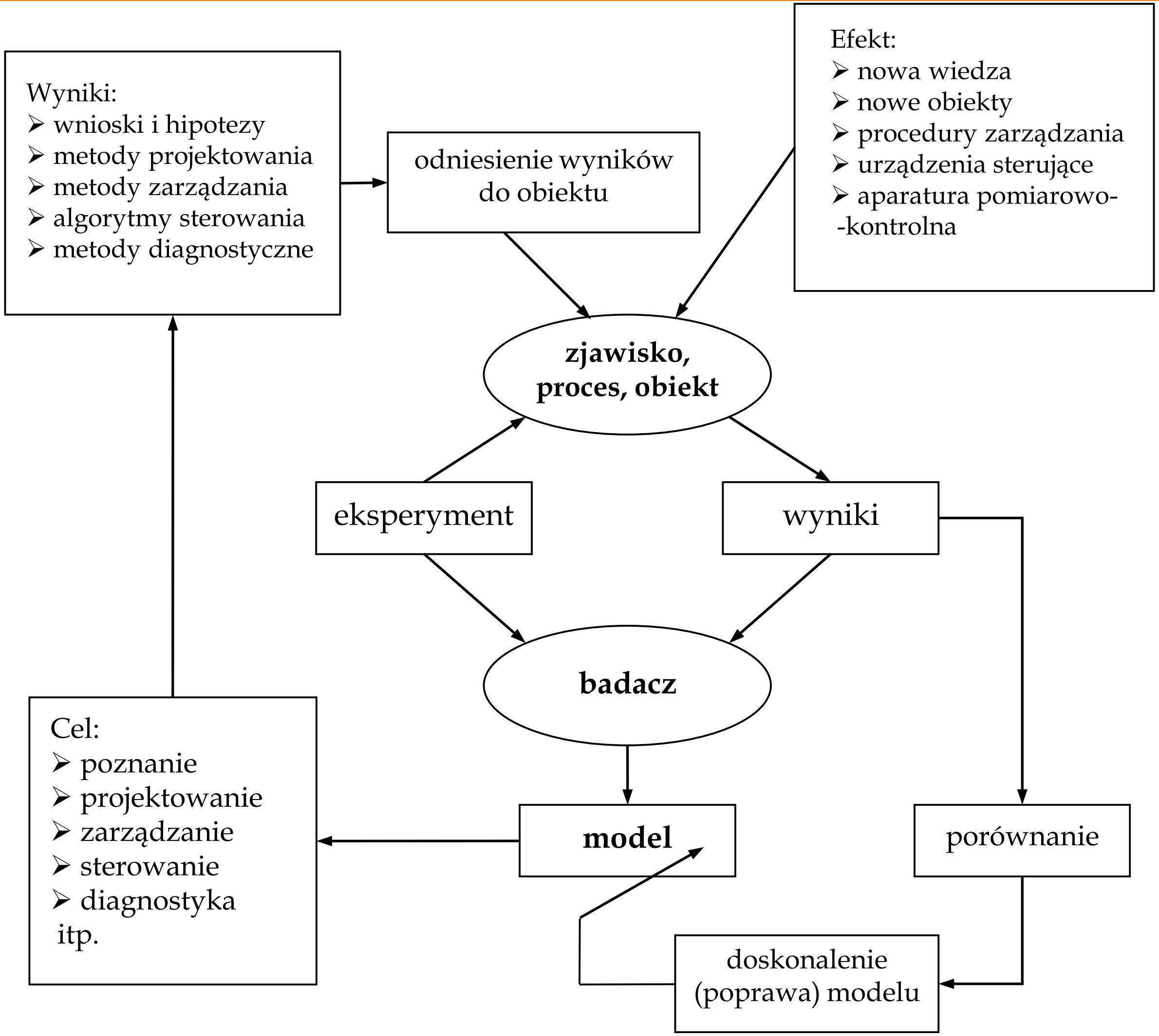
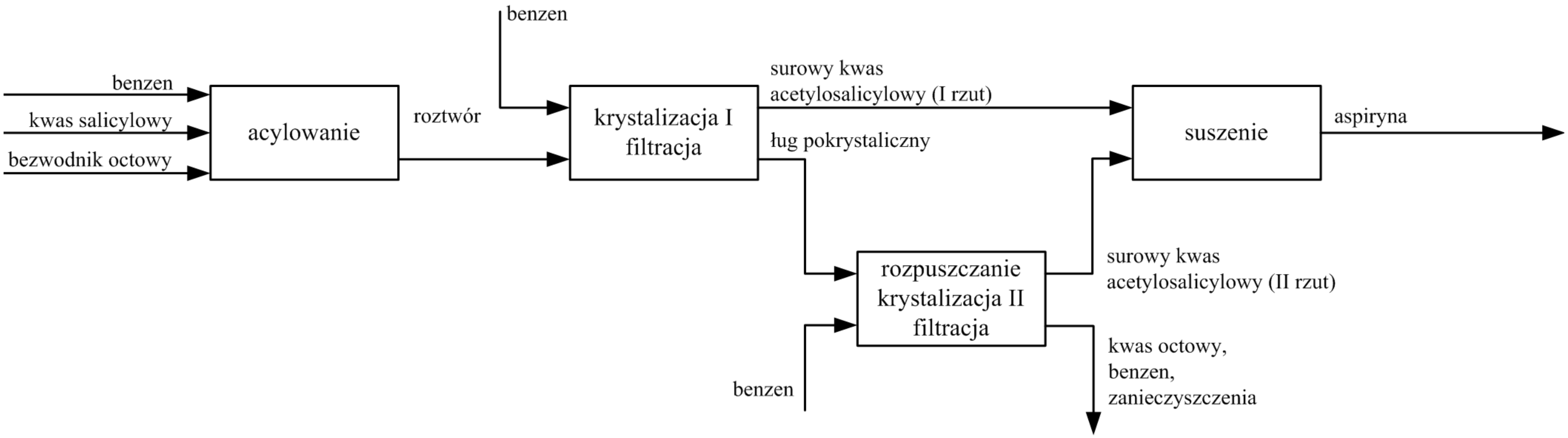
Metody Analizy Systemowej we Wspomaganiu Decyzji

Wykład 13. Wybrane zadania identyfikacji systemów złożonych

Model w badaniach systemowych



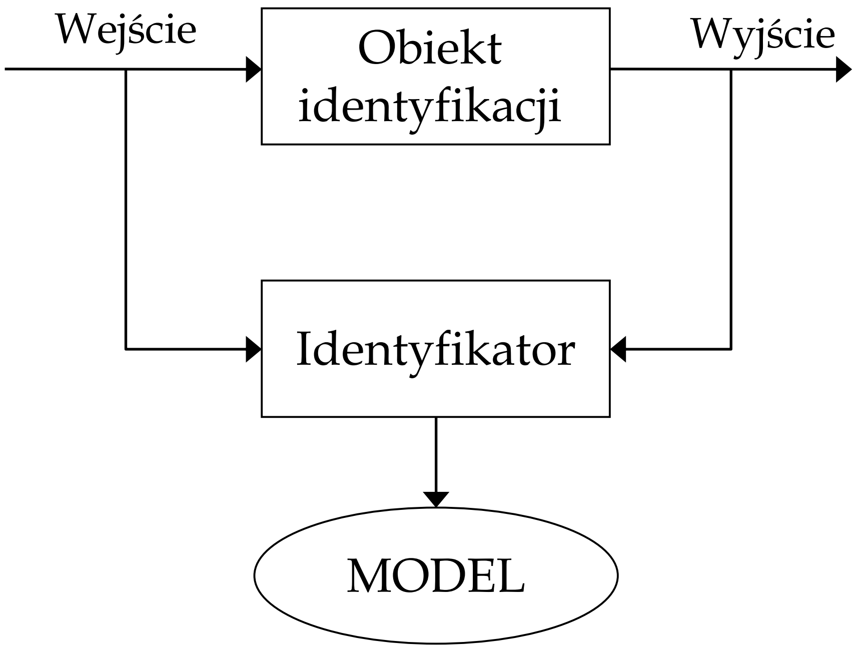
Elementy (komponenty) systemu
- działające części



System produkcji aspiryny

Zadanie identyfikacji

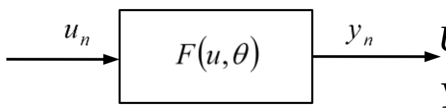
Zadanie identyfikacji – proces tworzenia modelu matematycznego obiektu na podstawie obiektu



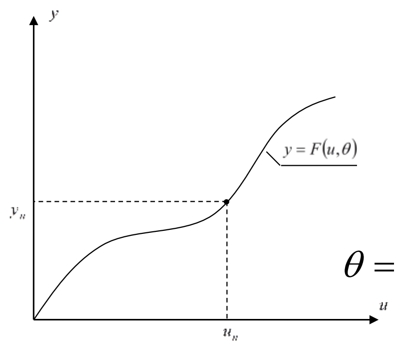
Podstawowe zadania identyfikacji podsumowanie

D e t e r m i n i s t y c z n y

Obiekt w klasie modeli

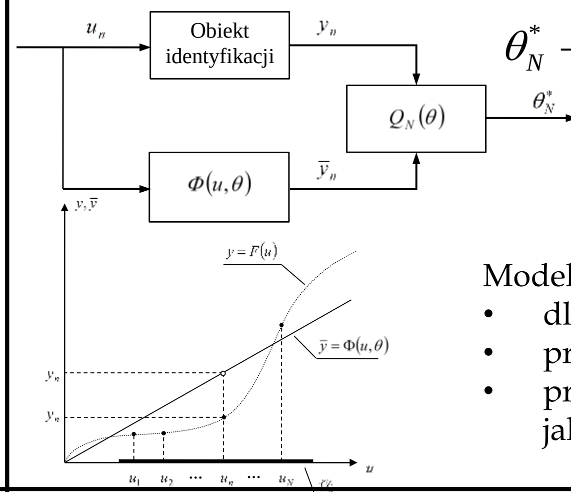


$$U_N = [u_1 \quad u_2 \quad \cdots \quad u_N],$$
$$Y_N = [y_1 \quad y_2 \quad \cdots \quad y_N]$$



$$y_n = F(u_n, \theta),$$
$$n = 1, 2, \ldots, N$$
$$Y_N = \overline{F}(U_N, \theta)$$
$$\theta = \overline{F}^{-1}(U_N, Y_N) \overset{df}{=} \Psi_N(U_N, Y_N)$$

Wybór optymalnego modelu

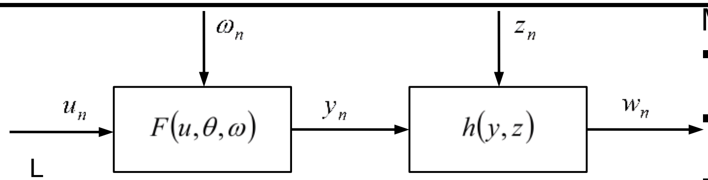


$$\bar{y} = \Phi(u, \theta_N^*)$$
$$\theta_N^* \to Q_N(\theta_N^*) = \min_{\theta} Q_N(\theta)$$
$$\theta_N^* = \Psi_N(U_N, Y_N)$$
$$U_N = [u_1 \quad u_2 \quad \cdots \quad u_N],$$
$$Y_N = [y_1 \quad y_2 \quad \cdots \quad y_N]$$

Model jest optymalny:

- dla zadanej serii pomiarowej
- przyjętego modelu
- przyjętego wskaźnika jakości identyfikacji

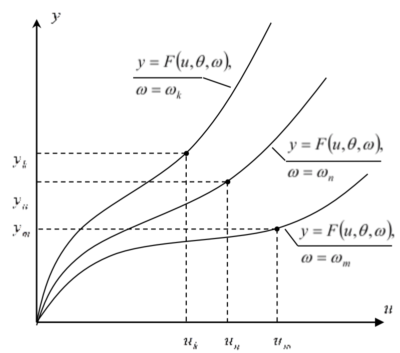
L o s o w y

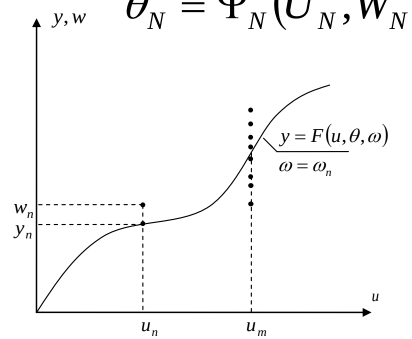


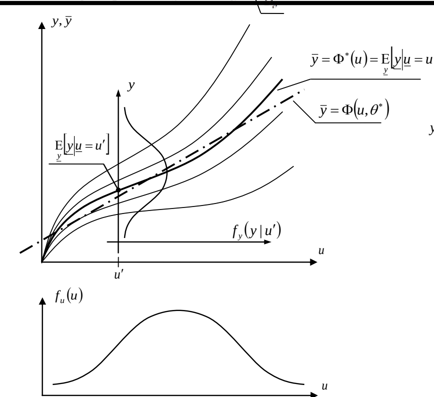
Metody:

- Najmniejszych kwadratów
- Maksymalnej wiarogodności
- Bayes'a

$$\theta_N = \Psi_N(U_N, W_N)$$

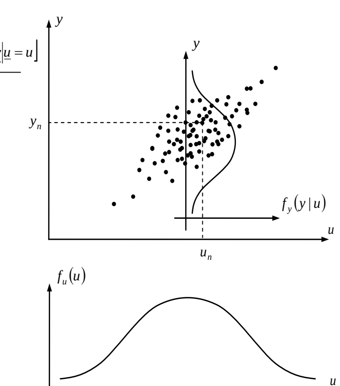






Pełna informacja

- Regresja I rodzaju
- Regresja II rodzaju



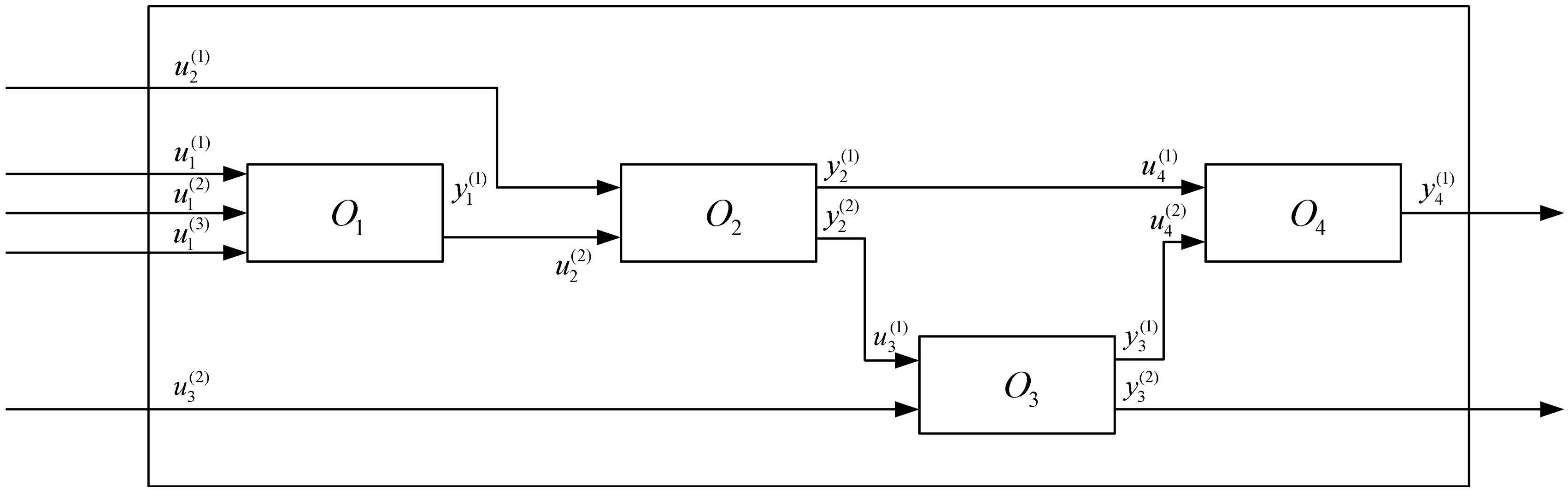
Niepełna informacja

- Estymacja wskaźnika jakości
- Estymacja parametrów rozkładu
- Estymacja rozkładu

Nowe problemy

- Opis systemu złożonego
- Identyfikacja przy ograniczonych możliwościach pomiarowych
- Identyfikacja lokalna i identyfikacja globalna
- Identyfikacja wielostopniowa

Elementy (komponenty) systemu
- działające części



Przykład systemu złożonego – produkcja aspiryny

Opis systemu złożonego

Wejściowo - wyjściowy system złożony z M podsystemami $O_1, O_2, \ldots, O_M$.

$y_m = F_m(u_m)$ (Uwaga! W tym miejscu mogą wystąpić różne typowe opisy)

Charakterystyka *m-tego* podsystemu, z wejściem u_m i wyjściem y_m, F_m jest znaną funkcją

$$u_m = \begin{bmatrix} u_m^{(1)} \\ u_m^{(2)} \\ \vdots \\ u_m^{(S_m)} \end{bmatrix} \in \mathscr{U}_m \subseteq \mathscr{R}^{S_m}, \quad y_m = \begin{bmatrix} y_m^{(1)} \\ y_m^{(2)} \\ \vdots \\ y_m^{(L_m)} \end{bmatrix} \in \mathscr{Y}_m \subseteq \mathscr{R}^{L_m}, \; m=1, 2, \ldots, M.$$

gdzie: S_m oraz L_m są odpowiednio wymiarami przestrzeni wejść wyjść,

Opis systemu złożonego

Niech *u*, *y*, oznacza odpowiednio wektory wszystkich wejść i wyjść systemu złożonego:

$$u = \begin{bmatrix} u^{(1)} \\ u^{(2)} \\ \vdots \\ u^{(S)} \end{bmatrix} \overset{df}{=} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_M \end{bmatrix}, \quad y = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(L)} \end{bmatrix} \overset{df}{=} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} \quad x = \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ \vdots \\ x^{(\tilde{S})} \end{bmatrix}$$

Gdzie, wektor wszystkich wejść systemu: $u \in \mathscr{U} = \mathscr{U}_1 \times \mathscr{U}_2 \times \cdots \times \mathscr{U}_M \subseteq \mathscr{R}^S, S = \sum_{m=1}^{M} S_m$,

wektor wszystkich wyjść systemu: $y \in \mathscr{Y} = \mathscr{Y}_1 \times \mathscr{Y}_2 \times \cdots \times \mathscr{Y}_M \subseteq \mathscr{R}^L, L = \sum_{m=1}^{M} L_m$,

oraz x jest $\tilde{S}$ - wymiarowym wejść zewnętrznych $x \in \mathscr{X} \subseteq \mathscr{U} \subseteq \mathscr{R}^{\tilde{S}}$.

Opis systemu złożonego

Struktura system jest dana zależnością:

$$u = Ay + Bx\ ,$$

gdzie: A jest $S \times L$ oraz B jest $S \times \widetilde{S}$ zero – jedynkową macierzą. Macierz A definiuje połączenia pomiędzy elementami system, tj.:

$$A = \left[a_{sl}\right]\Big|_{\substack{s=1,2,\ldots,S \\ l=1,2,\ldots,L}}\ , \quad a_{sl} = \begin{cases} 1 & if \quad u^{(s)} = y^{(l)} \\ 0 & if \quad u^{(s)} \neq y^{(l)} \end{cases},$$

A macierz B wskazuje wejścia zewnętrzne, tj.:

$$B = \left[b_{s\tilde{s}}\right]\Big|_{\substack{s=1,2,\ldots,S \\ \tilde{s}=1,2,\ldots,\widetilde{S}}}\ , \quad b_{s\tilde{s}} = \begin{cases} 1 & if \quad u^{(s)} = x^{(\tilde{s})} \\ 0 & if \quad u^{(s)} \neq x^{(\tilde{s})} \end{cases}.$$

Opis systemu złożonego

Wyjścia zewnętrzne systemu: $v = \begin{bmatrix} v^{(1)} \\ v^{(2)} \\ \vdots \\ v^{(\tilde{L})} \end{bmatrix}.$

$\tilde{L}$ wymiarowy wektor *v*, jest wektorem wybranych wyjść spośród wszystkich wyjść i jest określony przez $\tilde{L} \times L$ wymiarową macierz *C*,

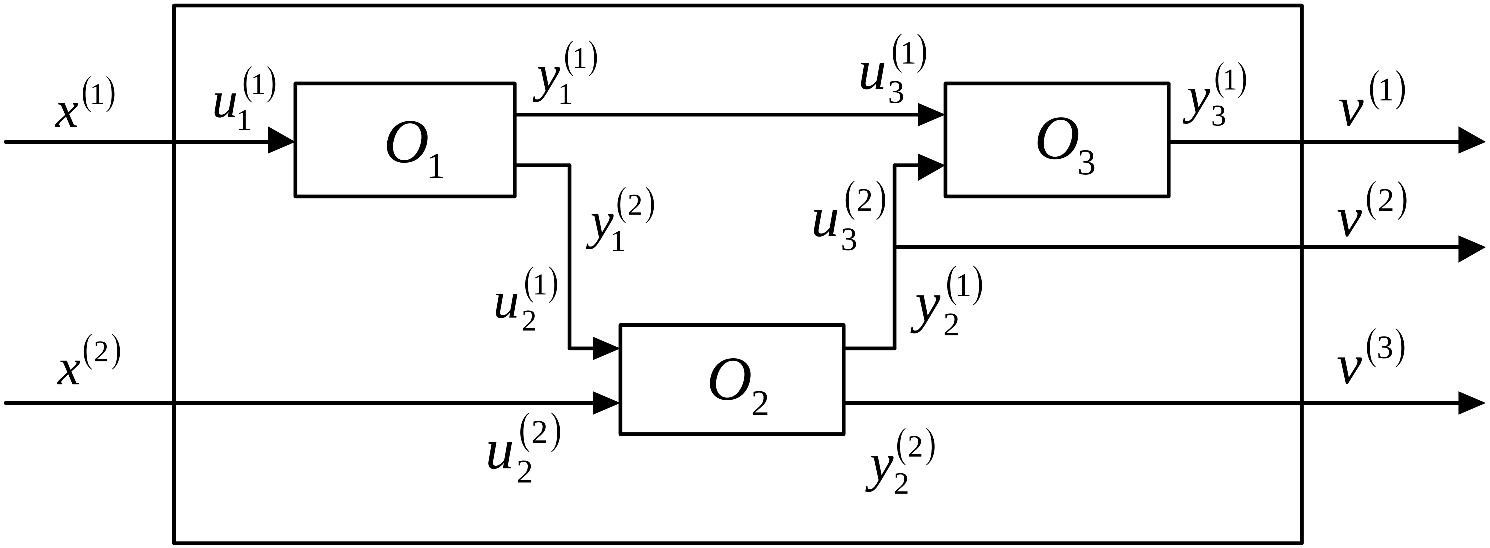
$$v = Cy ,$$

gdzie

$$C = \left[c_{\tilde{l}\,l}\right]\Big|_{\substack{\tilde{l}=1,2,\dots,\tilde{L} \\ l=1,2,\dots,L}}, \quad c_{\tilde{l}\,l} = \begin{cases} 1 & \text{if} \quad v^{(\tilde{l})} = y^{(l)} \\ 0 & \text{if} \quad v^{(\tilde{l})} \neq y^{(l)} \end{cases}.$$

Wektor wyjść zewnętrznych: $v \in \mathscr{V} = \{v : \forall\ y \in \mathscr{Y}, v = C\,y\} \subseteq \mathscr{R}^{\tilde{L}}$.

Opis systemu złożonego



Przykład systemu złożonego

Opis systemu złożonego

$$u = Ay + Bx$$

$$\begin{bmatrix} u_1^{(1)} \\ u_2^{(1)} \\ u_2^{(2)} \\ u_3^{(1)} \\ u_3^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1^{(1)} \\ y_1^{(2)} \\ y_2^{(1)} \\ y_2^{(2)} \\ y_3^{(1)} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix},$$

$$v = Cy$$

$$\begin{bmatrix} v^{(1)} \\ v^{(2)} \\ v^{(3)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_1^{(1)} \\ y_1^{(2)} \\ y_2^{(1)} \\ y_2^{(2)} \\ y_3^{(1)} \end{bmatrix}.$$

Opis systemu złożonego

Oznaczmy przez: $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} F_1(u_1) \\ F_2(u_2) \\ \vdots \\ F_M(u_M) \end{bmatrix} \stackrel{df}{=} \overline{F}(u)$.

$$y = \overline{F}(Ay + Bx) \Rightarrow y = \overline{F}^{-1}(x; A, B).$$

Rozwikłując powyższe równanie względem y otrzymujemy:

$$v = C\overline{F}^{-1}(x; A, B) = F(x)$$

opis systemu jako całość z wektorem wejść x oraz wektorem wyjść v.

Identyfikacja systemu złożonego przy ograniczonych możliwościach pomiaru

Rozważmy system złożony z M elementów $O_1, O_2, \ldots, O_M$. Struktura system złożonego określona jest przez macierze A i B. Charakterystyki statyczne znane są z dokładnością do parametrów:

$$y_m = F_m(u_m, \theta_m)$$

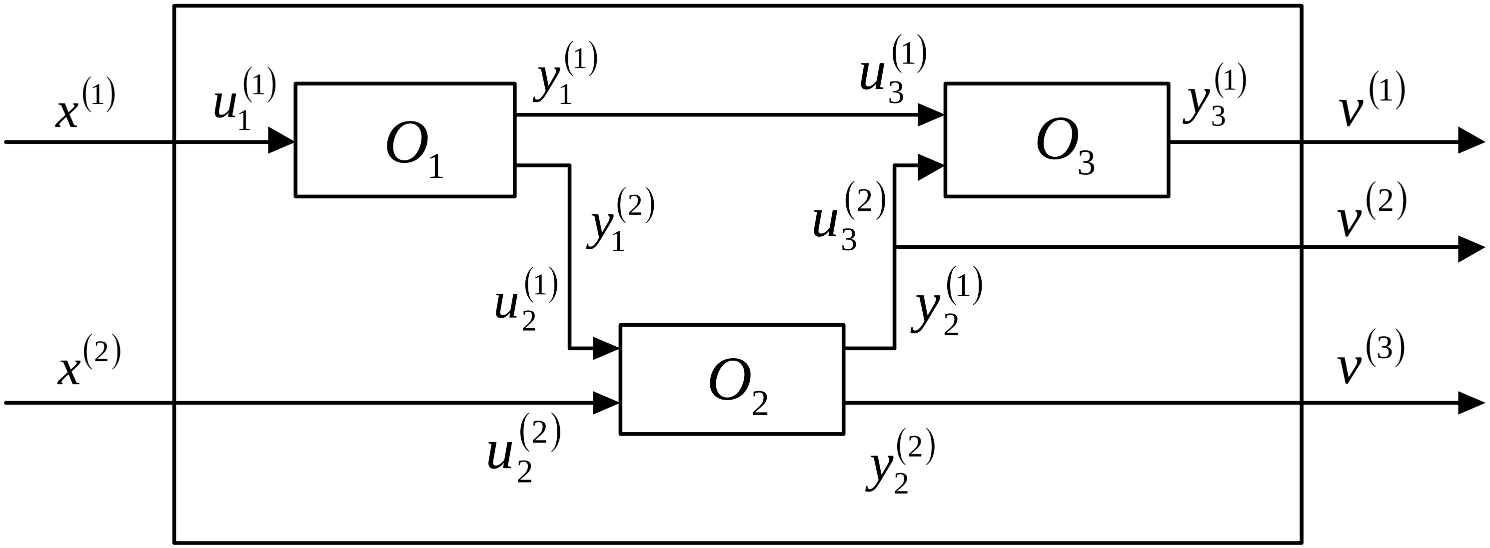
u_m oraz y_m są odpowiednio wejściem I wyjściem m – *tego* elementu, F_m jest znaną funkcją a θ_m jest R_m – wymiarowym wektorem nieznanych parametrów:

$$\theta_m = \begin{bmatrix} \theta_m^{(1)} \\ \theta_m^{(2)} \\ \vdots \\ \theta_m^{(R_m)} \end{bmatrix} \in \Theta_m \subseteq \mathscr{R}^{R_m}$$

Tylko wejścia zewnętrzne x oraz wybrane wyjścia v wskazane przez macierz C są mierzone.

Pojawia się pytanie: Czy możliwe jest jednoznaczne wyznaczenie nieznanych parametrów charakterystyki statycznej obiektu na podstawie ograniczonych możliwości pomiarowych?

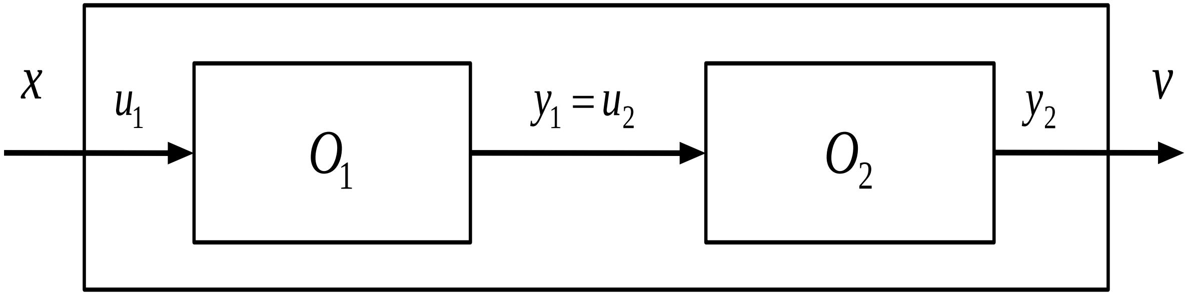
Opis systemu złożonego



Przykład systemu złożonego

Identyfikacja systemu złożonego przy ograniczonych możliwościach pomiaru

Następujący przykład ilustruje problem.



System o strukturze szeregowej

Powyższy system opisujemy następującym układem równań:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x = \begin{bmatrix} x \\ y_1 \end{bmatrix},$$

$$v = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = y_2 .$$

Identyfikacja systemu złożonego przy ograniczonych możliwościach pomiaru

Example 1 Let static characteristics of the first and second element are:

$$y_1 = u_1^{\theta_1}, \qquad y_2 = \theta_2 u_2 .$$

The system as a new element has the form: $v = \theta_2 \, x^{\theta_1} = e^{\theta_1 \, x + \ln \theta_2}$, where $\theta^T = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}$ is a vector of unknown parameters of complex system characteristic.

For external inputs $x_1 > 0, x_2 > 0$, $x_1 \neq x_2$ outputs v_1 and v_2 were measured ($N = 2$).

Now the system description has the form: $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \theta_2 \, x_1^{\theta_1} \\ \theta_2 \, x_2^{\theta_1} \end{bmatrix} = \begin{bmatrix} e^{\theta_1 \, x_1 + \ln \theta_2} \\ e^{\theta_1 \, x_2 + \ln \theta_2} \end{bmatrix}$,

and identification algorithm:

$$\begin{bmatrix} \theta_1 \\ \ln \theta_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\ln v_2 - \ln v_1}{\ln x_2 - \ln x_1} \\ \dfrac{\ln v_1 \ln x_2 - \ln v_2 \ln x_1}{\ln x_2 - \ln x_1} \end{bmatrix}.$$

Identification of complex systems with restricted measurement possibilities

Przykład 2. Załóżmy, że obydwa elementy system są liniowe,

$$y_1 = \theta_1 u_1 \qquad y_2 = \theta_2 u_2 \,.$$

Opis nowego system jako całość ma postać:

$$v = \theta_1 \theta_2 x \,,$$

$\theta^T = [\theta_1 \quad \theta_2]$ jest wektorem nieznanych parametrów.

Dla zewnętrznych $x_1 \neq x_2$ zmierzono wyjścia v_1 oraz v_2 ($N = 2$).

Podstawiając wyniki pomiarów do opisu system mamy: $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \theta_1 \theta_2 x_1 \\ \theta_1 \theta_2 x_2 \end{bmatrix}$.

Z powyższego możemy wyznaczyć jedynie: $\theta_1 \theta_2 = \dfrac{v_n}{x_n}, n = 1, 2..$

Deterministic separability

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} F_1(u_1, \theta_1) \\ F_2(u_2, \theta_2) \\ \vdots \\ F_M(u_M, \theta_M) \end{bmatrix} \stackrel{df}{=} \overline{F}(u, \theta) \, ,$$

gdzie θ jest wektorem wszystkich parametrów elementów system tj.:

$$\theta = \begin{bmatrix} \theta^{(1)} \\ \theta^{(2)} \\ \vdots \\ \theta^{(R)} \end{bmatrix} \stackrel{df}{=} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_M \end{bmatrix}, \quad \theta \in \Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_M \subseteq \mathscr{R}^R, \, R = \sum_{m=1}^{M} R_m \, .$$

Charakterystyka system jako całość z wejściami zewnętrznymi x oraz wyjściami v jest następująca:

$$v = C\overline{F}^{-1}(x, \theta; A, B) \stackrel{df}{=} F(x, \theta) \, .$$

Deterministic separability

Przykład 3. Niech opis *m – tego* elementu ma postać:

$$y_m = \Xi_m u_m\,,\ m = 1, 2, \ldots, M$$, gdzie: Ξ_m is $L_m \times S_m$ macierz parametrów tj.:

$$\Xi_m = \left[\theta_m^{(l,s)}\right]\Big|_{\substack{l=1,2,\ldots,L_m \\ s=1,2,\ldots,S_m}} .$$

Obecnie opis systemu jako całość ma postać:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} \Xi_1 & O & \cdots & O \\ O & \Xi_2 & \cdots & O \\ \vdots & \vdots & \ddots & \vdots \\ O & O & \cdots & \Xi_M \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_M \end{bmatrix}, \quad \Xi \overset{df}{=} \begin{bmatrix} \Xi_1 & O & \cdots & O \\ O & \Xi_2 & \cdots & O \\ \vdots & \vdots & \ddots & \vdots \\ O & O & \cdots & \Xi_M \end{bmatrix}$$

Deterministic separability

$$y = \Xi u = \Xi\left[Ay + Bx\right] \Rightarrow y = \left(I - \Xi A\right)^{-1} \Xi Bx\,.$$

Taking into account system structure and measurement possibilities the description of the whole system has the form:

$$v = C\left(I - \Xi\, A\right)^{-1} \Xi\, B\, x\,,$$

under condition that $\left(I - \Xi\, A\right)$ is non-singular matrix. Notice that complex system composed by linear elements gives linear system

$$v = \tilde{\Xi}\, x\,,$$

where:

$$\tilde{\Xi} = C\left(I - \Xi\, A\right)^{-1} \Xi\, B\,.$$

Deterministic separability

Definition 2 The complex system with a given structure and characteristics of each element known with accuracy to parameters is called separable, if the element defined by measurement possibilities is identifiable.

Using Definition of the identifiably we can conclude, that complex system is separable if there exists such a sequence

$$X_N = \begin{bmatrix} x_1 & x_2 & \cdots & x_N \end{bmatrix},$$

which together with corresponding results of output measurements

$$V_N = \begin{bmatrix} v_1 & v_2 & \cdots & v_N \end{bmatrix},$$

uniquely determines plant characteristic parameters. In the other words, the complex system is separable if there exists such an identification sequence X_N , which together with output measurements V_N gives system of equations

$$v_n = F(x_n, \theta), \quad n = 1, 2, \ldots, N ,$$

for which there exists the unique solution with respect to θ .

Deterministic separability

Let us notice that parameters θ in the characteristic, for the newly defined element, are transformed. The characteristic can be rewritten in the form:

$$v = C\overline{F}^{-1}(x,\theta;A,B) = F(x,\theta) \stackrel{df}{=} \widetilde{F}(x,\widetilde{\theta}),$$

and finally:

$$v = \widetilde{F}(x,\widetilde{\theta}),$$

where vector of plant parameters $\widetilde{\theta}$ in the newly defined plant is given by the relation

$$\widetilde{\theta} \stackrel{df}{=} \Gamma(\theta),$$

where Γ is a known function such that:

$$\Gamma : \Theta \to \widetilde{\Theta}, \widetilde{\Theta} = \left\{\widetilde{\theta} : \forall\, \theta \in \Theta, \widetilde{\theta} = \Gamma(\theta)\right\} \subseteq \mathscr{R}^{\widetilde{R}},$$

$\widetilde{R}$ is dimension of the new plant characteristic and $\widetilde{F}$ is a known function, such that:

$$\widetilde{F} : \mathscr{X} \times \widetilde{\Theta} \to \mathscr{V}.$$

Deterministic separability

The form of functions $\widetilde{F}$ and Γ depends on the description of particular elements, system structure and measurement possibilities. Coming back to the examples, the characteristics for Example 1 has the form:

$$v = \theta_2 x^{\theta_1} = e^{\theta_1 x + \ln\theta_2} = e^{\widetilde{\theta}_1 x + \widetilde{\theta}_2},$$

where $\widetilde{\theta} = \begin{bmatrix} \widetilde{\theta}_1 \\ \widetilde{\theta}_2 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \ln\theta_2 \end{bmatrix}$, and characteristic for Example 2:

$$v = \theta_1 \theta_2 x = \widetilde{\theta} x,$$

where $\widetilde{\theta} = \theta_1 \theta_2$.

Deterministic separability

Theorem 1 The complex system is separable if the element is identifiable and function Γ is an one to one mapping.

Proof:
$$v_n = \tilde{F}(x_n, \tilde{\theta}), \quad n = 1, 2, \ldots, N ,$$

which have the unique solution with respect to $\tilde{\theta}$. The system of equations may be rewritten in the form:

$$V_N = \tilde{\bar{F}}\left(X_N, \tilde{\theta}\right).$$

and solution with respect to $\tilde{\theta}$ gives identification algorithm:

$$\tilde{\theta} = \tilde{\bar{F}}^{-1}\left(X_N, V_N\right) \stackrel{df}{=} \tilde{\Psi}_N\left(X_N, V_N\right),$$
$$\theta = \Gamma^{-1}\left(\tilde{\theta}\right),$$

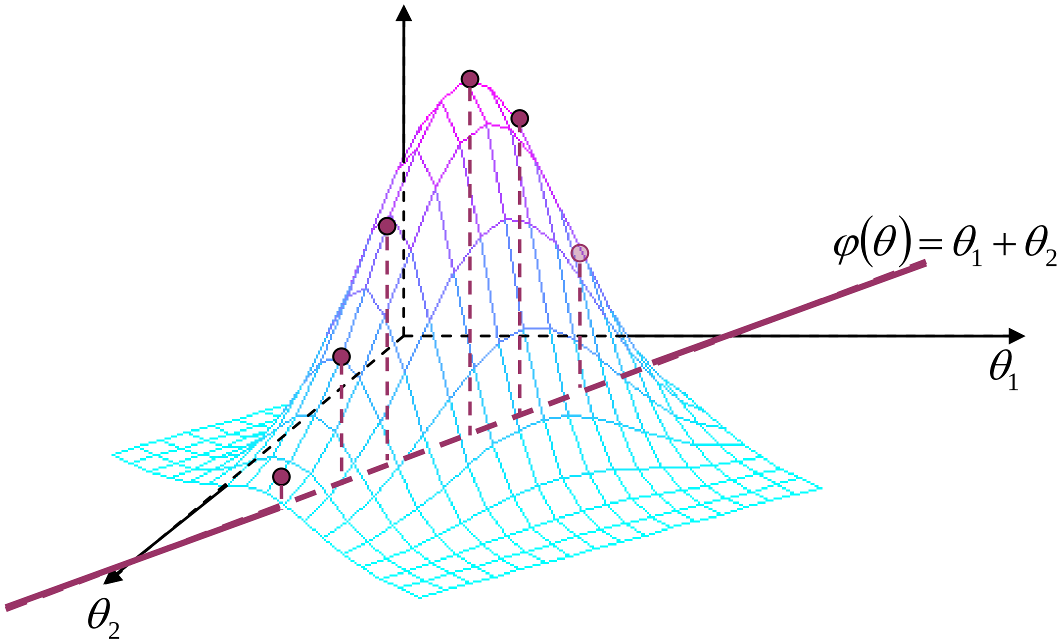
where Γ^{-1} is an inverse function of Γ . Finally, we obtain identification algorithm:

$$\theta = \Gamma^{-1}\left(\tilde{\Psi}_N\left(X_N, Y_N\right)\right) = \Psi_N\left(X_N, Y_N\right).$$

Probabilistic separability

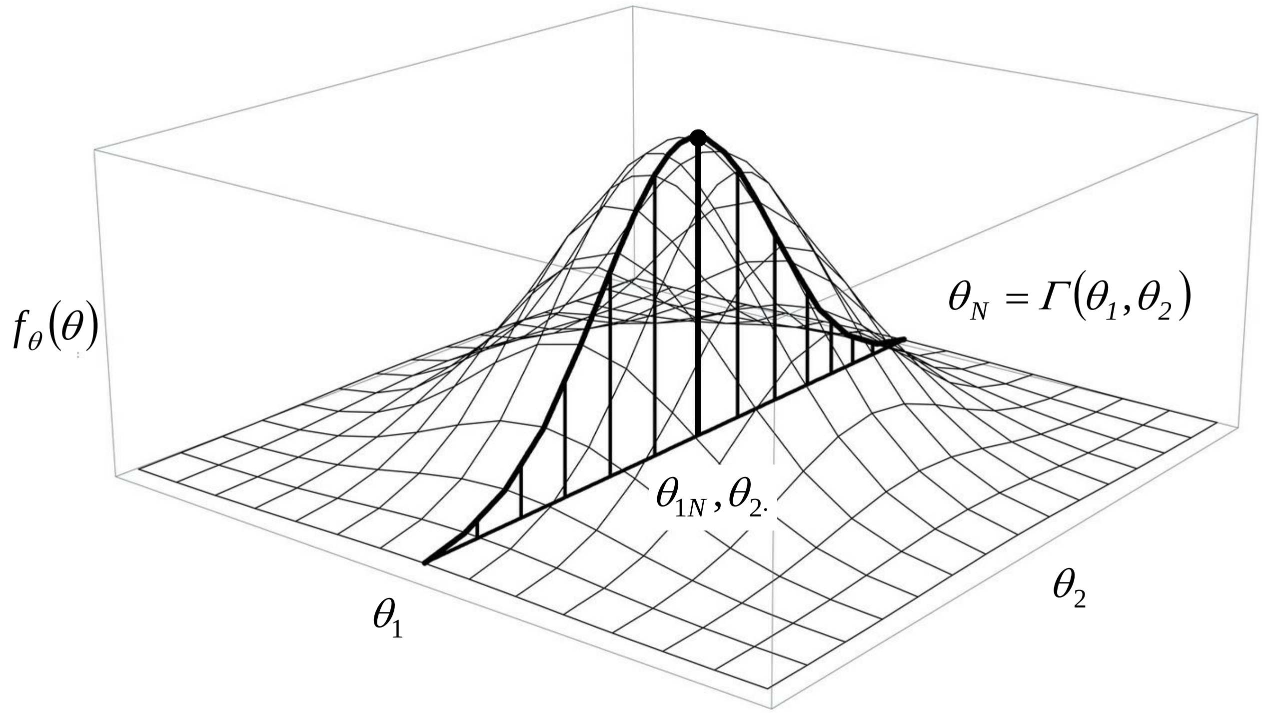
θ_1, θ_2

$f_1(\theta_1), f_2(\theta_2)$



Probabilistic separability

utilization of a′priori information



$$v = C\overline{F}_y^{-1}(x,\theta;A,B) = F(x,\theta) \stackrel{\mathrm{df}}{=} \widetilde{F}(x,\widetilde{\theta}),\ \widetilde{\theta} \stackrel{\mathrm{df}}{=} \Gamma(\theta),$$

Choice of the best model of complex system

Let us consider input - output complex system with *M* elements $O_1, O_2, ..., O_M$. The structure of the complex system, are given by matrices *A* and *B* in complex system description. Static characteristic for elements is unknown. For *m-th* element with input u_m and output y_m the following model is proposed:

$$\bar{y}_m = \Phi_m(u_m, \theta_m),$$

$\bar{y}_m$ is output of the model, Φ_m is a known, proposed by us, function and θ_m is vector of unknown parameters of the *m-th* element model. Model output and vector of model parameters are elements of the respective spaces, i.e.:

$$\bar{y}_m = \begin{bmatrix} \bar{y}_m^{(1)} \\ \bar{y}_m^{(2)} \\ \vdots \\ \bar{y}_m^{(S_m)} \end{bmatrix} \in \mathscr{Y}_m \subseteq \mathscr{R}^{L_m}, \ \theta_m = \begin{bmatrix} \theta_m^{(1)} \\ \theta_m^{(2)} \\ \vdots \\ \theta_m^{(R_m)} \end{bmatrix} \in \Theta_m \subseteq \mathscr{R}^{R_m},$$

Choice of the best model of complex system

Let: $u = \begin{bmatrix} u^{(1)} \\ u^{(2)} \\ \vdots \\ u^{(S)} \end{bmatrix} \overset{df}{=} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_M \end{bmatrix}, \quad y = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(L)} \end{bmatrix} \overset{df}{=} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix}, \quad \bar{y} = \begin{bmatrix} \bar{y}^{(1)} \\ \bar{y}^{(2)} \\ \vdots \\ \bar{y}^{(L)} \end{bmatrix} \overset{df}{=} \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \vdots \\ \bar{y}_M \end{bmatrix},$

where vector of all the system inputs: $u \in \mathscr{U} = \mathscr{U}_1 \times \mathscr{U}_2 \times \cdots \times \mathscr{U}_M \subseteq \mathscr{R}^S, S = \sum_{m=1}^{M} S_m$, and vector of all the plant outputs and all model outputs: $y, \bar{y} \in \mathscr{Y} = \mathscr{Y}_1 \times \mathscr{Y}_2 \times \cdots \times \mathscr{Y}_M \subseteq \mathscr{R}^L, L = \sum_{m=1}^{M} L_m$. Only some outputs will be taken into account. Those outputs will be called the global outputs v,and they are shown by $\tilde{L} \times L$ dimensional matrices C where $\tilde{L}$ is a number of selected outputs from the all outputs of complex system, i.e.:

$$v = Cy,$$

where $v \in \mathscr{V} = \{v : \forall\ y \in \mathscr{Y}, v = C\,y\} \subseteq \mathscr{R}^{\tilde{L}}$.

Choice of the best model of complex system

$$\bar{y} = \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \vdots \\ \bar{y}_M \end{bmatrix} = \begin{bmatrix} \Phi_1(u_1, \theta_1) \\ \Phi_2(u_2, \theta_2) \\ \vdots \\ \Phi_M(u_M, \theta_M) \end{bmatrix} \overset{df}{=} \overline{\Phi}(u, \theta),$$

$$u = A\bar{y} + Bx,$$

$$\bar{v} = C\,\bar{y},$$

where: $\bar{v} \in \mathscr{V} = \{\bar{v} : \forall\ \bar{y} \in \mathscr{Y}, \bar{v} = C\,\bar{y}\} \subseteq \mathscr{R}^{\tilde{L}}$,

and unknown vector of model parameters: $\theta \in \Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_M \subseteq \mathscr{R}^R,\ R = \sum_{m=1}^{M} R_m$.

Choice of the best model of complex system

Output of the model may be expressed as:

$$\bar{y} = \overline{\Phi}(A\bar{y} + Bx, \theta) .$$

Solution of above equation with respect to $\bar{y}$ gives:

$$\bar{y} = \overline{\Phi}^{-1}(x, \theta; A, B) .$$

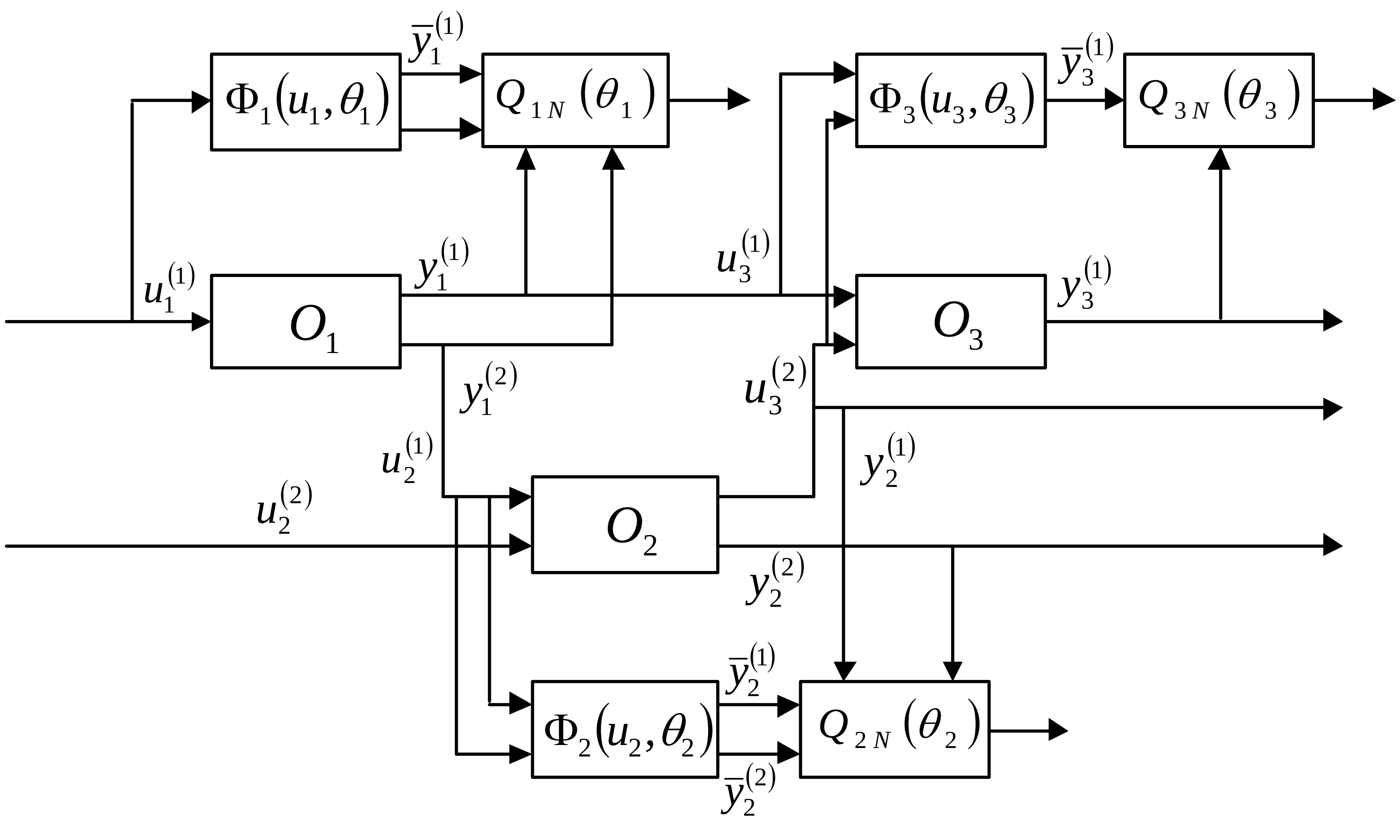
and finally by substituting this solution into the system description we obtain:

$$\bar{v} = C\overline{\Phi}^{-1}(x, \theta; A, B) \overset{df}{=} \Phi(x, \theta) .$$

The relation above is a model of the complex system with external input x and global output $\bar{v}$.

Choice of the best model of complex system

- Locally optimal model of complex system



Choice of the best model of complex system

- Locally optimal model of complex system

Now, it will be assumed that each element of complex system is observed independently. For *m-th* elements for a given input sequence the output is measured. The results of the experiment are collected in the following matrices:

$$U_{mN_m} = \left[u_{m1} \quad u_{m2} \quad \cdots \quad u_{mN_m}\right], \quad Y_{mN_m} = \left[y_{m1} \quad y_{m2} \quad \cdots \quad y_{mN_m}\right],$$

where N_m is a number of measurement points for *m-th* element, $m = 1, 2, \ldots, M$.

For each *m-th* element we propose a model. We also propose the performance index:

$$Q_{mN_m}(\theta_m) = \left\| Y_{mN_m} - \overline{Y}_{mN_m}(\theta_m) \right\|_{U_{mN_m}} ,$$

where: $\overline{Y}_{mN_m}(\theta_m) \overset{df}{=} \left[\Phi_m(u_{m1}, \theta_m) \quad \Phi_m(u_{m2}, \theta_m) \quad \cdots \quad \Phi_m\left(u_{mN_m}, \theta_m\right)\right].$

Choice of the best model of complex system

- Locally optimal model of complex system

The example of performance indexes $Q_{N_m m}(\theta_m)$:

$$Q_{N_m m}(\theta_m) = \sum_{n=1}^{N_m} q_m(y_{mn}, \bar{y}_{mn}) = \sum_{n=1}^{N_m} q_m(y_{mn}, \Phi_m(u_{mn}, \theta_m)),$$

$$Q_{mN_m}(\theta_m) = \max_{1 \le n \le N_m} \{q_m(y_{mn}, \bar{y}_{mn})\} = \max_{1 \le n \le N_m} \{q_m(y_{mn}, \Phi_m(u_{mn}, \theta))\}.$$

Choice of the best model of complex system

- Locally optimal model of complex system

The optimal value of vector model parameters for *m-th* element is obtained by minimization of the performance index $Q_{mN_m}(\theta_m)$ with respect to θ_m from the space Θ_m

$$\theta^*_{mN_m} \rightarrow \quad Q_{mN_m}\left(\theta^*_{mN_m}\right) = \min_{\theta_m \in \Theta_m} Q_{mN_m}(\theta_m),$$

where $\theta^*_{mN_m}$ is the optimal value of *m-th* model parameters and function Φ_m with vector $\theta^*_{mN_m}$, i.e.:

$$\bar{y}_m = \Phi_m(u_m, \theta^*_{mN_m}),$$

is called locally optimal model of *m-th* element. The local identification task is repeated for each element separately, i.e.: $m = 1, 2, \ldots, M$.

Choice of the best model of complex system

- Locally optimal model of complex system

Let us denote vector of all the locally optimal parameters by: $\theta_N^* \overset{df}{=} \begin{bmatrix} \theta_{1N_1}^* \\ \theta_{2N_2}^* \\ \vdots \\ \theta_{MN_M}^* \end{bmatrix}$,

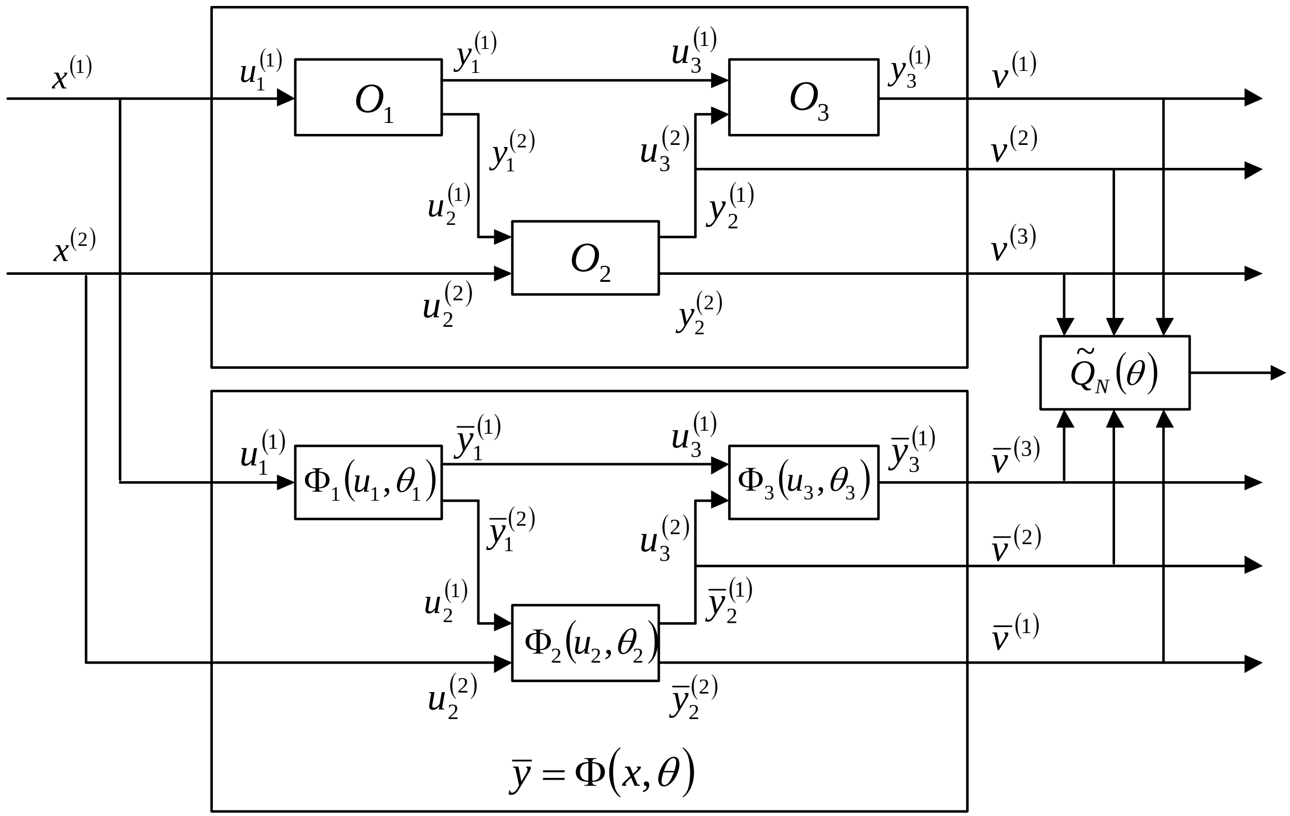
where: $N = \sum_{m=1}^{M} N_m$. The model of the complex system with locally optimal parameters, i.e.:

$$\bar{v} = C\overline{\Phi}^{-1}(x, \theta_N^*; A, B) \overset{df}{=} \Phi(x, \theta_N^*) .$$

is called locally optimal model of complex system.

Choice of the best model of complex system

Globally optimal model of complex system



Choice of the best model of complex system

- Globally optimal model of complex system

Performance index: $$Q_N(\theta) = \left\| V_N - \overline{V}_N(\theta) \right\|_{X_N}$$

shows the difference between the result of the experiment V_N and the respective sequence of model outputs calculated for input sequence X_N , i.e.: $\overline{V}_N(\theta) \overset{df}{=} \left[\Phi(x_1, \theta) \quad \Phi(x_2, \theta) \quad \cdots \quad \Phi(x_N, \theta) \right]$.

$$\widetilde{\theta}_N \rightarrow \quad Q_N\left(\widetilde{\theta}_N\right) = \min_{\theta \in \Theta} Q_N(\theta),$$

where: $\widetilde{\theta}_N$ is the optimal vector of model parameters and

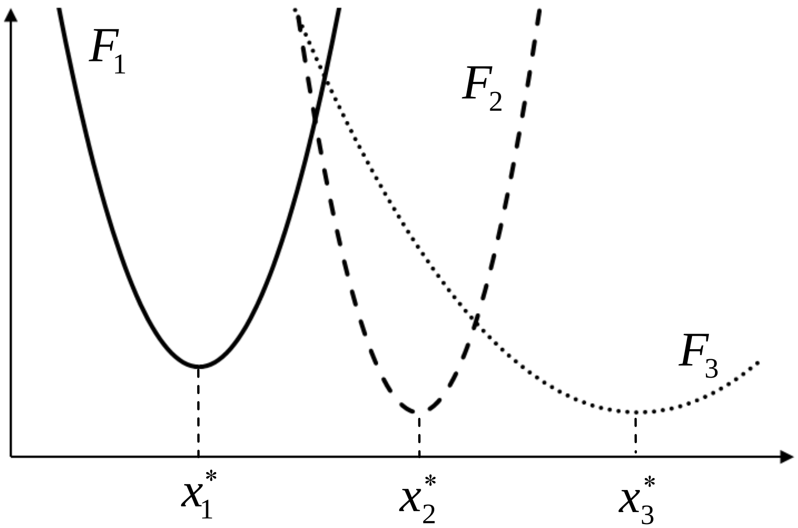
$$\overline{v} = \Phi(u, \widetilde{\theta}_N)$$

is called a globally optimal model of complex system.

Multi-criteria approach

x – vector of decision variables

$F_1(x), F_2(x), \ldots, F_M(x)$ – performance indices



Polioptymalizacja

Syntetyczny wskaźnik jakości

$$F(x) = H\left(F_1(x), F_2(x), \ldots, F_K(x)\right)$$

H(.) – funkcja monotoniczna ze względu na każdą składową

np.: $$F(x) = \sum_{k=1}^{K} \alpha_k F_k(x)$$

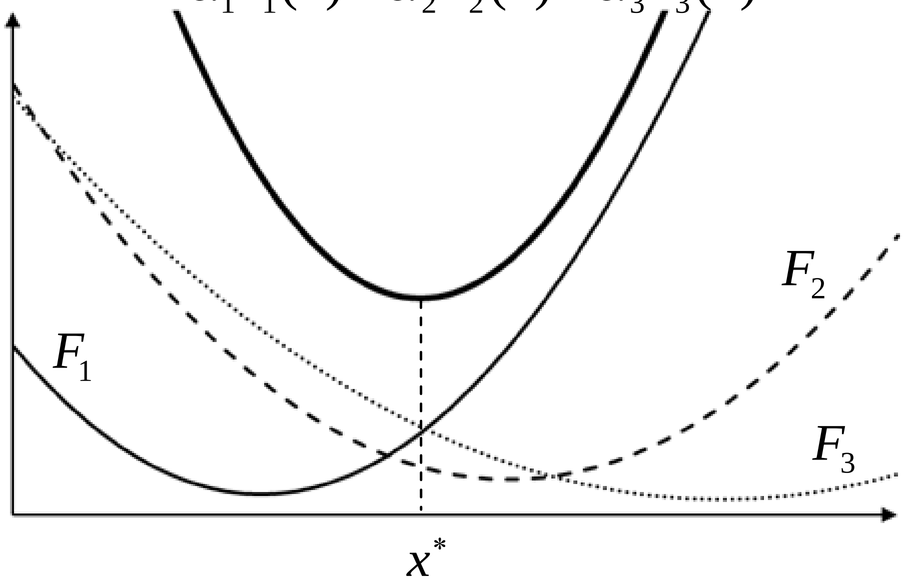
gdzie:

$$\sum_{k=1}^{K} \alpha_k = 1, \;\; \alpha_k > 0, \;\; k = 1,2,\ldots,K$$

$$F(x) = \prod_{k=1}^{K} F_k(x)$$

$$x^* \to F(x^*) = \min_{x \in \mathcal{D}_x} F(x)$$

$$F = \alpha_1 F_1(x) + \alpha_2 F_2(x) + \alpha_3 F_3(x)$$



Polioptymalizacja

Optymalizujemy wybrany wskaźnik,

Pozostałe wskaźniki spełnione są w sposób zadowalający.

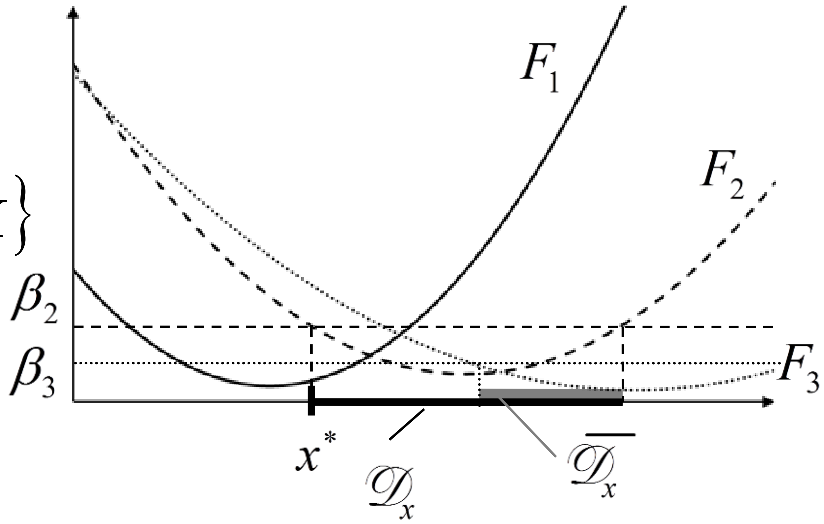
Niech $F_1(x)$ - wybrany wskaźnik

$$F_k(x) \le \beta_k, \quad k = 2,3,\dots,K$$

Wskaźniki spełnione w sposób zadowalający

$$\overline{\mathscr{D}_x} = \mathscr{D}_x \cap \left\{ x \in \mathscr{R}^S : F_k(x) \le \beta_k,\ k = 2,\dots,K \right\}$$

$$x^* \to F_1(x^*) = \min_{x \in \overline{\mathscr{D}_x}} F_1(x)$$



Choice of the best model of complex system

- Globally optimal model with local quality guaranteed

Synthetic performance index which takes into account both local and global model qualities:

$$\overline{Q}_N(\theta) = \alpha_0 Q_N(\theta) + \sum_{m=1}^{M} \alpha_m Q_{mN}(\theta_m)\,,$$

where: $\alpha_0, \alpha_1, \cdots, \alpha_M$ is a sequence of weight coefficients. They show weigh of participation of global and local performance indexes respectively, in the synthetic performance index. Now the optimal model parameters for synthetic performance index:

$$\overline{\theta}_N \rightarrow \overline{Q}_N(\overline{\theta}_N) = \min_{\theta \in \Theta} \overline{Q}_N(\theta)\,,$$

where $\overline{\theta}_N$ is an optimal vector for global model for synthetic performance index.

Choice of the best model of complex system

- Globally optimal model with local quality guaranteed

In the other approach we assume that local models must be sufficiently good:

$$Q_{mN}(\theta_m) \le \beta_m, \quad m = 1, 2, \cdots, M \ ,$$

where quality sufficient number β_m is grater then locally optimal performance index, i.e.:

$$\beta_m > Q_{mN}(\theta_m^*), \quad m = 1, 2, \cdots, M \, .$$

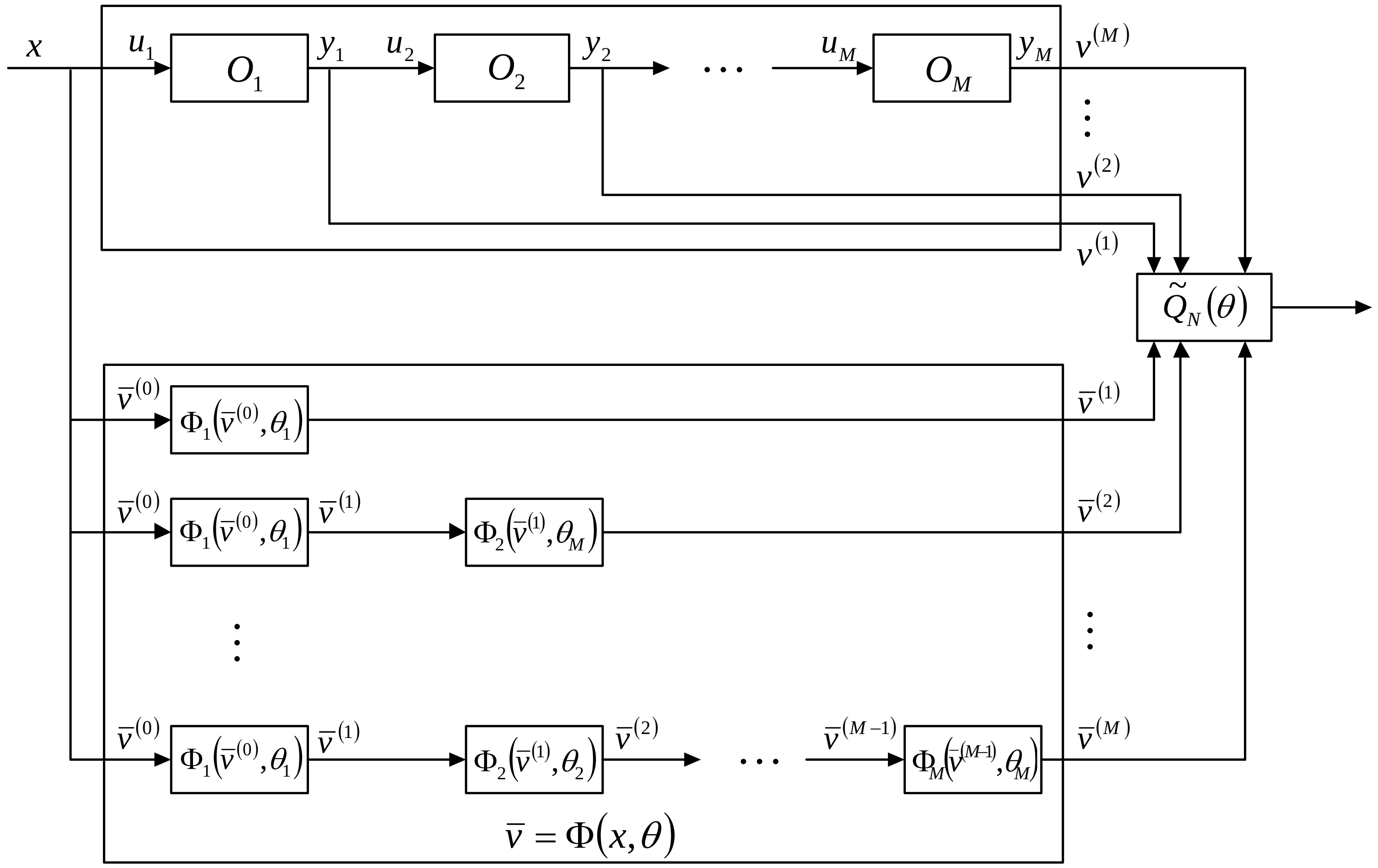
Now, the optimal model parameters will be obtained by minimization global performance index with additional constrains, i.e.:

$$\widetilde{\theta}_N^* \to \quad Q_N\left(\widetilde{\theta}_N^*\right) = \min_{\theta \in \widetilde{\Theta}} Q_N(\theta),$$

where $\widetilde{\Theta} \overset{df}{=} \left\{\theta \in \Theta \subseteq \mathscr{R}^R : \quad Q_m(\theta_m) \le \beta_m, \quad \beta_m > Q_m(\theta_m^*), \quad m = 1, 2, \cdots, M \, .\right\}$

and $\widetilde{\theta}_N^*$ is a globally optimal vector parameters sufficiently good for local models.

Complex system with cascade structure



Complex system with cascade structure

The global model has the form:

$$\begin{bmatrix} \bar{v}^{(1)} \\ \bar{v}^{(2)} \\ \vdots \\ \bar{v}^{(M)} \end{bmatrix} = \begin{bmatrix} \Phi_1(x,\theta_1) \\ \Phi_2(\Phi_1(x,\theta_1),\theta_2) \\ \vdots \\ \Phi_M(\cdots\Phi_2(\Phi_1(x,\theta_1),\theta_2)\cdots\theta_M) \end{bmatrix}$$

Complex system with cascade structure

Notice that the model may be given in the recursive fom:

$$\bar{v}^{(m+1)} = \Phi_m(\bar{v}^{(m)}, \theta_m), \quad m = 0, 1, \cdots, M$$

where $\bar{v}^{(0)} = x$.

The global identification performance index is:

$$Q(\theta) = \sum_{n=1}^{N} \sum_{m=1}^{M} q(v_n^{(m)}, \bar{v}_n^{(m)})$$

Identification algorithm based on dynamic programming

Step 1. Determine $\tilde{a}_M$ such that

$$\tilde{a}_M = \Psi_M\left(V_N^{(M)}, \overline{V}_N^{(M-1)}\right) \rightarrow \min_{a_M} \sum_{n=1}^{N} q_M\left(v_n^{(M)}, \Phi_M\left(\overline{v}_n^{(M-1)}, a_M\right)\right) = \overline{Q}_M\left(V_N^{(M)}, \overline{V}_N^{(M-1)}\right)$$

where:

$V_N^{(M)} = \left[v_1^{(M)}\ v_2^{(M)} \cdots v_N^{(M)}\right]$ - sequence of measurements of M–th global output,

$\overline{V}_N^{(M-1)}$ - sequence of outputs of (M-1)-th element in cascade structure.

$$\overline{V}_N^{(M-1)} = \left[\overline{v}_1^{(M-1)}\ \overline{v}_2^{(M-1)} \cdots \overline{v}_N^{(M-1)}\right]$$

$$\overline{V}_N^{(M-1)} = \left[\Phi_{M-1}(\overline{v}_1^{(M-2)}, a_{M-1})\ \Phi_{M-1}(\overline{v}_2^{(M-2)}, a_{M-1}) \cdots \Phi_{M-1}(\overline{v}_N^{(M-2)}, a_{M-1})\ \right] = \overline{\Phi}_{M-1}(\overline{V}_N^{(M-2)}, a_{M-1})$$

Consequently solution may be rewritten:

$$\overline{Q}_M\left(V_N^{(M)}, \overline{V}_N^{(M-1)}\right) = \overline{Q}_M\left(V_N^{(M)}, \overline{\Phi}_{M-1}(\overline{V}_N^{(M-2)}, a_{M-1})\right)$$

Identification algorithm based on dynamic programming

Step 2. Determine $\tilde{a}_{M-1}$ such that

$$\tilde{a}_{M-1} = \Psi_{M-1}\left(V_N^{(M)}, V_N^{(M-1)}, \overline{V}_N^{(M-2)}\right) \rightarrow$$

$$\min_{a_{M-1}} \left\{ \sum_{n=1}^{N} q_{M-1}\left(v_n^{(M-1)}, \Phi_{M-1}\left(\overline{v}_n^{(M-2)}, a_{M-1}\right)\right) + \overline{Q}_M\left(V_N^{(M)}, \overline{\Phi}_M(\overline{V}_N^{(M-2)}, a_{M-1})\right)\right\} = \overline{Q}_{M-1}\left(V_N^{(M)}, V_N^{(M-1)}, \overline{V}_N^{(M-2)}\right)$$

where:

$V_N^{(M-1)} = \left[v_1^{(M-1)}\ v_2^{(M-1)} \cdots v_N^{(M-1)}\right]$ - sequence of measurements of (M-1)–th global output,

$\overline{V}_N^{(M-2)}$ - sequence of outputs of (M-2)-th element in cascade structure.

$$\overline{V}_N^{(M-2)} = \left[\overline{v}_1^{(M-2)}\ \overline{v}_2^{(M-2)} \cdots \overline{v}_N^{(M-2)}\right]$$

We obtain $\overline{V}_N^{(M-2)} = \left[\Phi_{M-2}(\overline{v}_1^{(M-3)}, a_{M-2})\ \Phi_{M-2}(\overline{v}_2^{(M-3)}, a_{M-2}) \cdots \Phi_{M-2}(\overline{v}_N^{(M-3)}, a_{M-2})\right] = \overline{\Phi}_{M-2}(\overline{V}_N^{(M-3)}, a_{M-2})$

Consequently solution may be rewritten

$$\overline{Q}_{M-1}\left(V_N^{(M)}, V_N^{(M-1)}, \overline{V}_N^{(M-2)}\right) = \overline{Q}_{M-1}\left(V_N^{(M)}, V_N^{(M-1)}, \overline{\Phi}_{M-2}(\overline{V}_N^{(M-3)}, a_{M-2})\right)$$

Identification algorithm based on dynamic programming

Step (M-1). Determine $\tilde{a}_2$ such that

$$\tilde{a}_2 = \Psi_2\left(V_N^{(M)}, V_N^{(M-1)}, \cdots, V_N^{(2)}, \overline{V}_N^{(1)}\right) \quad \rightarrow$$

$$\min_{a_2}\left\{\sum_{n=1}^{N} q_2\left(v_n^{(2)}, \Phi_2\left(\overline{v}_n^{(1)}, a_2\right)\right) + \overline{Q}_3\left(V_N^{(M)}, V_N^{(M-1)}, \cdots, V_N^{(3)}, \overline{\Phi}_2(\overline{V}_N^{(1)}, a_2)\right)\right\} = \overline{Q}_2\left(V_N^{(M)}, V_N^{(M-1)}, \cdots, V_N^{(2)}, \overline{V}_N^{(1)}\right)$$

where:

$V_N^{(2)} = \left[v_1^{(2)}\ v_2^{(2)} \cdots v_N^{(2)}\right]$ - sequence of measurements of second global output,

$\overline{V}_N^{(1)}$ - sequence of outputs of the first element in cascade structure.

$\overline{V}_N^{(1)} = \left[\overline{v}_1^{(1)}\ \overline{v}_2^{(1)} \cdots \overline{v}_N^{(1)}\right]$

We obtain $\overline{V}_N^{(1)} = \left[\Phi_1(\overline{v}_1^{(0)}, a_1)\ \ \Phi_1(\overline{v}_2^{(0)}, a_1) \cdots \Phi_1(\overline{v}_N^{(0)}, a_1)\right] = \left[\Phi_1(x_1, a_1)\ \ \Phi_1(x_2, a_1) \cdots \Phi_1(x_N, a_1)\right] = \overline{\Phi}_1(X_N, a_1)$

where: $\overline{V}_N^{(0)} = \left[\overline{v}_1^{(0)}\ \overline{v}_2^{(0)} \cdots \overline{v}_N^{(0)}\right] = \left[x_1\ x_2 \cdots x_N\right] = X_N$, X_N - sequence of the external input.

Consequently:

$$\overline{Q}_2\left(V_N^{(M)}, V_N^{(M-1)}, \cdots, V_N^{(2)}, \overline{V}_N^{(1)}\right) = \overline{Q}_2\left(V_N^{(M)}, V_N^{(M-1)}, \cdots V_N^{(2)}, \overline{\Phi}_1(X_N, a_1)\right)$$

Identification algorithm based on dynamic programming

Step M. Determine $\tilde{a}_1$ such that

$$\tilde{a}_1 = \Psi_1\left(V_N^{(M)}, V_N^{(M-1)}, \cdots, V_N^{(1)}, X_N\right) \rightarrow$$

$$\min_{a_1}\left\{\sum_{n=1}^{N} q_1\left(v_n^{(1)}, \Phi_2\left(x_n, a_1\right)\right) + \overline{Q}_2\left(V_N^{(M)}, V_N^{(M-1)}, \cdots, V_N^{(2)}, \overline{\Phi}_2(X_N, a_1)\right)\right\} = \overline{Q}_1\left(V_N^{(M)}, V_N^{(M-1)}, \cdots, V_N^{(1)}, X_N\right)$$

where: $V_N^{(1)} = \left[v_1^{(1)}\ v_2^{(1)} \cdots v_N^{(1)}\right]$ - sequence of measurements of first global output.

Identification algorithm based on dynamic programming

Now we can came back and determine:

$$\tilde{a}_1 = \Psi_1\left(V_N^{(M)}, V_N^{(M-1)}, \cdots, V_N^{(1)}, X_N\right)$$

$$\overline{V}_N^{(1)} = \overline{\Phi}_1(X_N, \tilde{a}_1) = \overline{\Phi}_1\left(X_N, \Psi_1\left(V_N^{(M)}, V_N^{(M-1)}, \cdots, V_N^{(1)}, X_N\right)\right)$$

which is necessary to determine

$$\tilde{a}_2 = \Psi_2\left(V_N^{(M)}, V_N^{(M-1)}, \cdots, V_N^{(2)}, \overline{V}_N^{(1)}\right) = \Psi_2\left(V_N^{(M)}, V_N^{(M-1)}, \cdots, V_N^{(2)}, \overline{\Phi}_1\left(X_N, \Psi_1\left(V_N^{(M)}, V_N^{(M-1)}, \cdots, V_N^{(1)}, X_N\right)\right)\right);$$

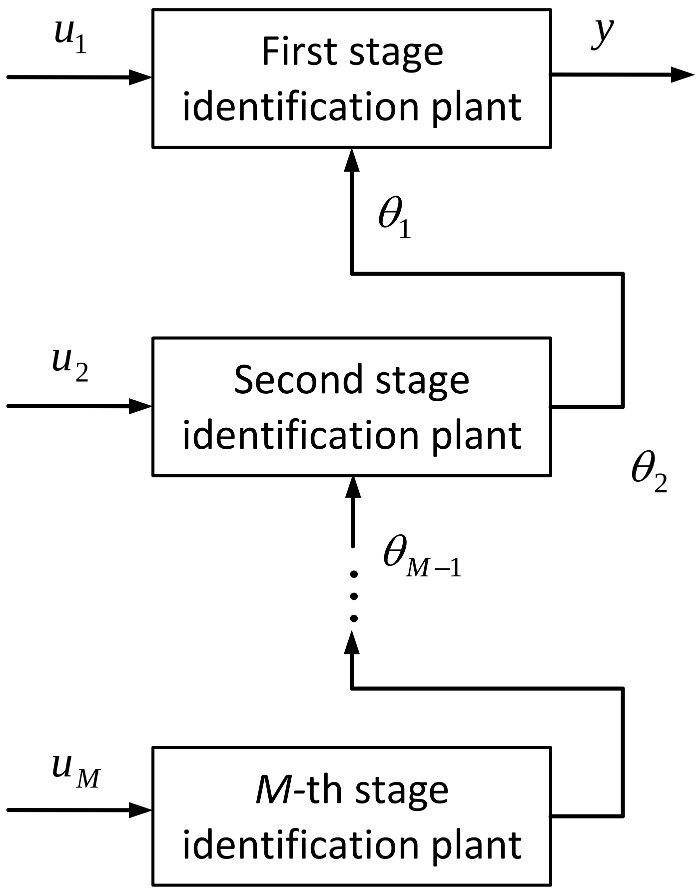
Finally

$$\tilde{a}_M = \Psi_M\left(V_N^{(M)}, \overline{V}_N^{(M-1)}\right)$$

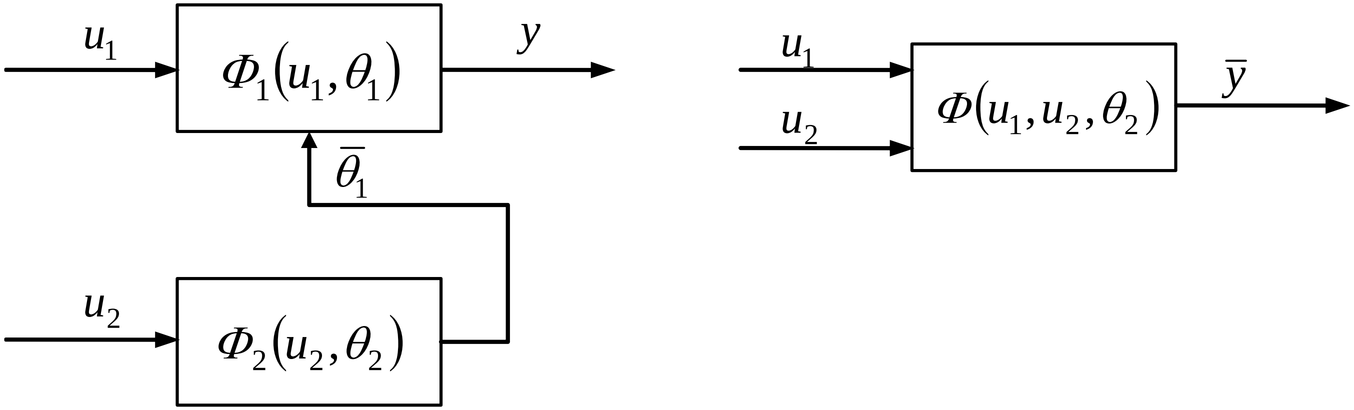
the sequence will be determined at the previous step as

$$\overline{V}_N^{(M-1)} = \overline{\Phi}_{M-1}(\overline{V}_N^{(M-2)}, \tilde{a}_{M-1}) \; .$$

Two stage identification and it's applications



Two stage identification and it's applications



$$\bar{y} = \Phi(u_1, u_2, \theta_2) \overset{\text{df}}{=} \Phi_1(u_1, \Phi_2(u_2, \theta_2))$$

Two stage identification and it's applications

Measurements: $U_{1N_1n_2} \overset{\text{df}}{=} \begin{bmatrix} u_{11n_2} & u_{12n_2} & \cdots & u_{1N_1n_2} \end{bmatrix}$, $\quad Y_{N_1n_2} \overset{\text{df}}{=} \begin{bmatrix} y_{1n_2} & y_{2n_2} & \cdots & y_{N_1n_2} \end{bmatrix}$,

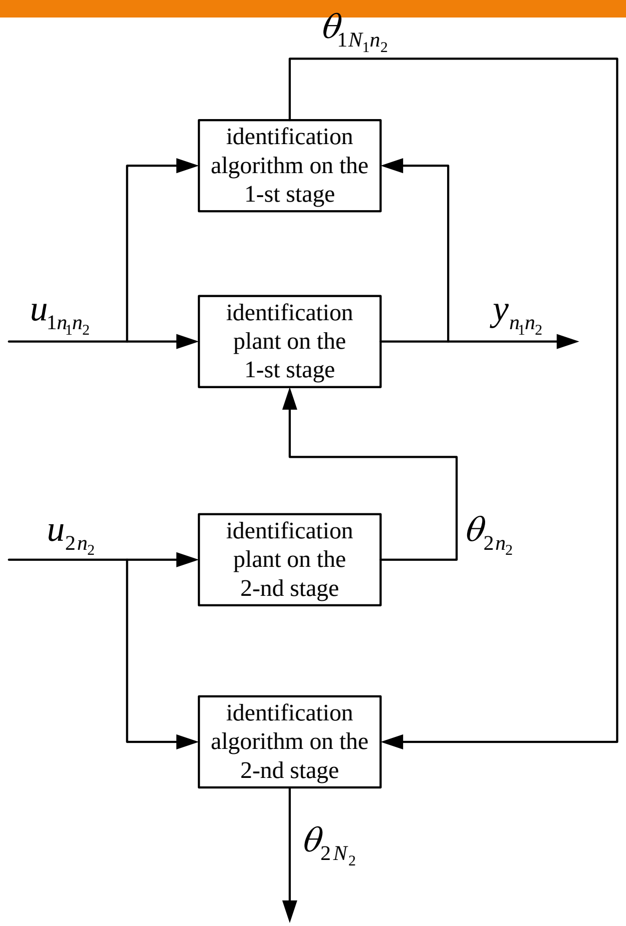
Performance indices: $Q_{1N_1n_2}(\theta_1) = \frac{1}{N_1} \sum_{n_1=1}^{N_1} q_1\left(y_{n_1n_2}, \Phi_1\left(u_{1n_1n_2}, \theta_1\right)\right)$

$$\theta^*_{2N_2} = \Psi_{2N_2}\left(U_{2N_2}, \Xi^*_{1N_1N_2}\right) \qquad \theta^*_{1N_1n_2} = \Psi_{1N_1}\left(U_{1N_1n_2}, Y_{N_1n_2}\right)$$

$$\Xi^*_{1N_1N_2} \overset{\text{df}}{=} \begin{bmatrix} \theta^*_{1N_11} & \theta^*_{1N_12} & \cdots & \theta^*_{1N_1N_2} \end{bmatrix}.$$

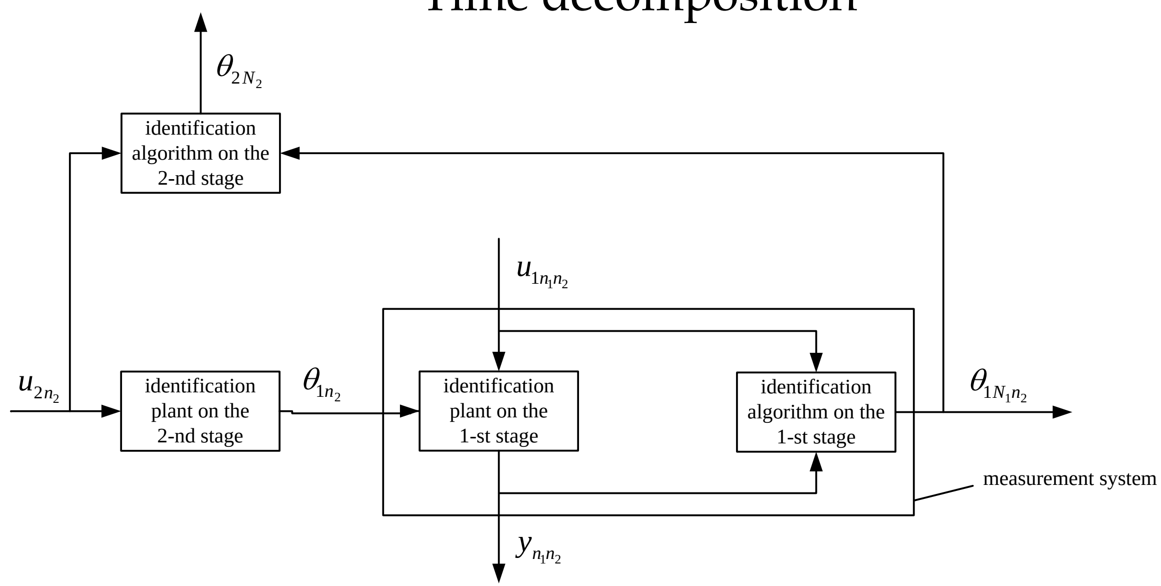
$$Q_{2N_2}(\theta_2) = \frac{1}{N_2} \sum_{n_2=1}^{N_2} q_2\left(\theta^*_{1N_1n_2}, \Phi_2\left(u_{2n_2}, \theta_2\right)\right) \qquad Q_{N_1N_2}(\theta_2) = \frac{1}{N_1N_2} \sum_{n_2=1}^{N_2} \sum_{n_1=1}^{N_1} q_1\left(y_{n_1n_2}, \Phi\left(u_{1n_1n_2}, u_{2n_2}, \theta_2\right)\right)$$

Two stage identification and it's applications



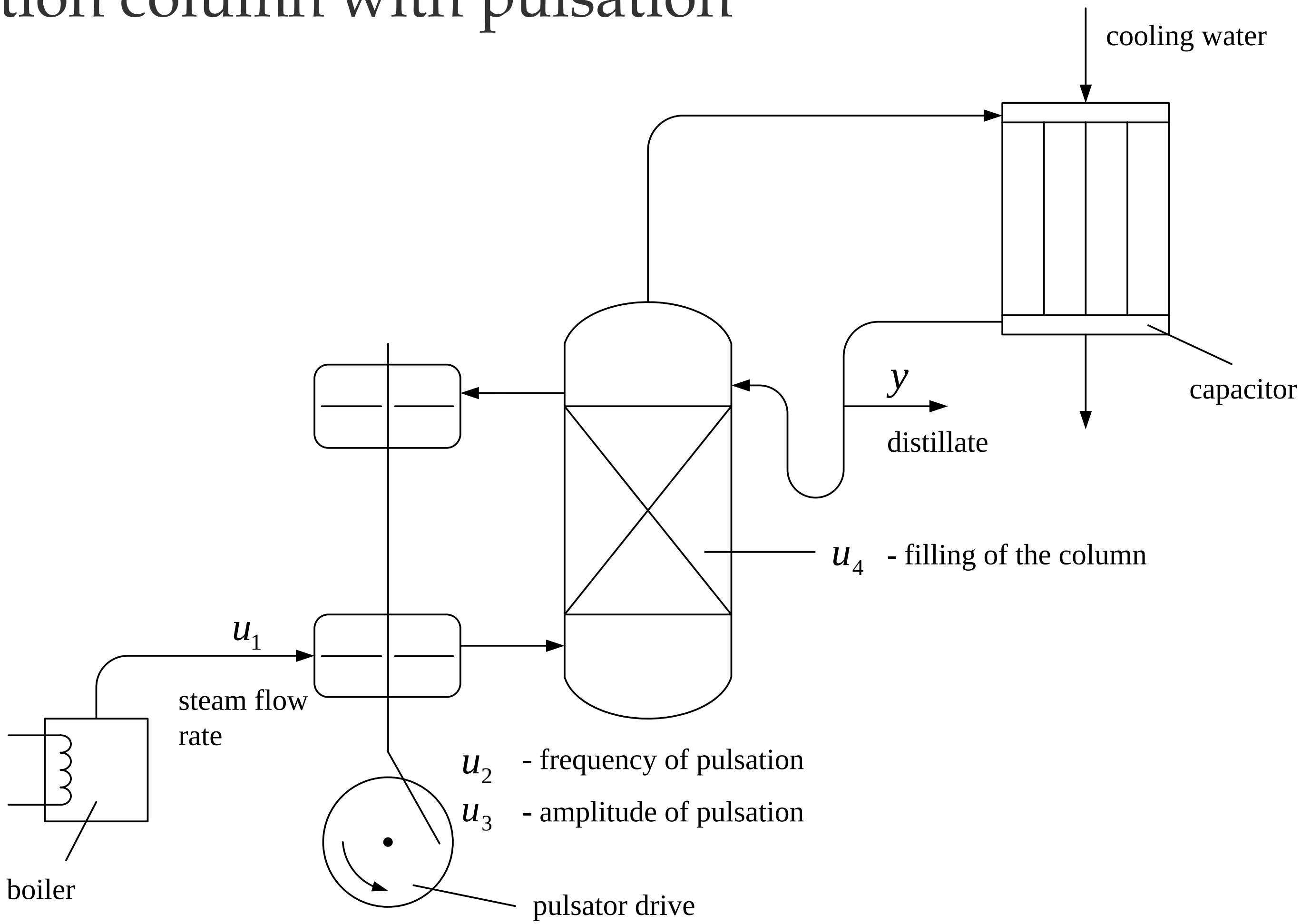
Two stage identification

- Space decomposition
- Time decomposition



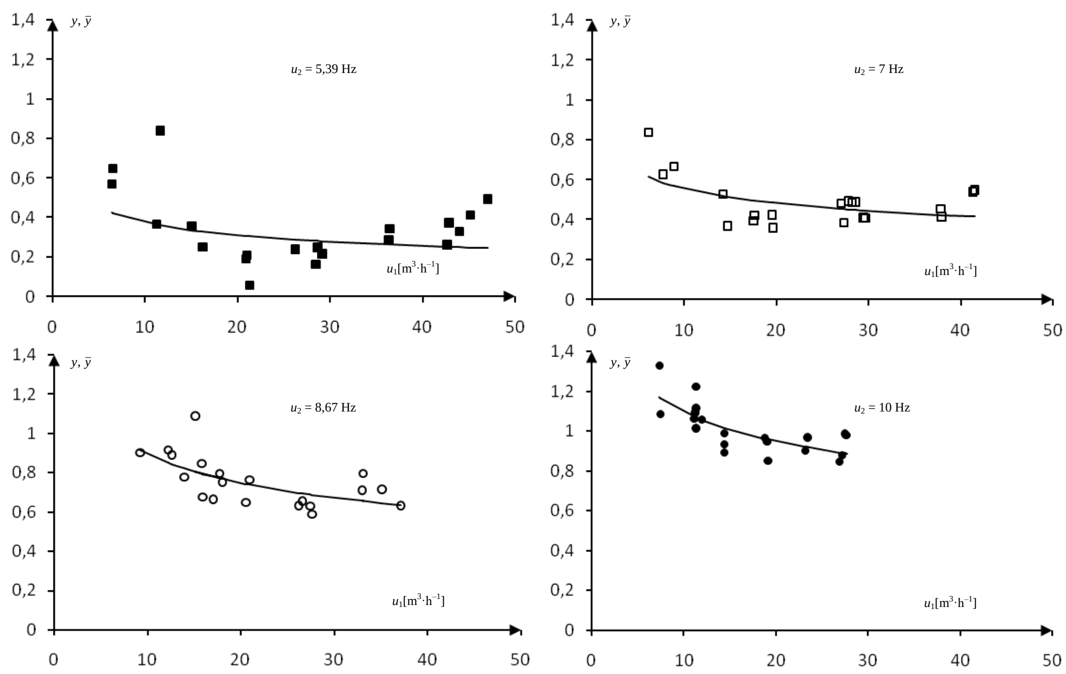
Two stage identification and it's applications

- Distillation column with pulsation



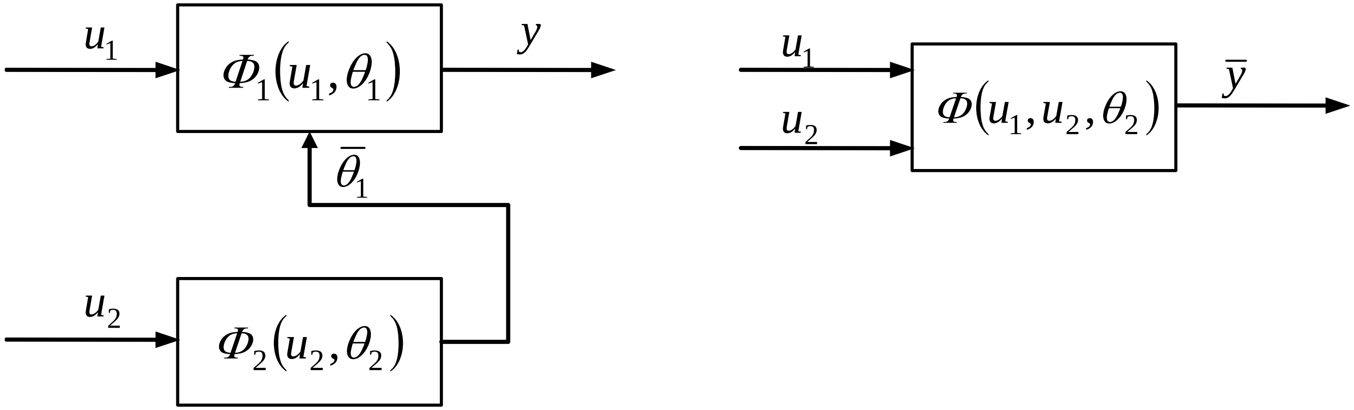
Two stage identification and it's applications

- Distillation column with pulsation



Two stage identification and it's applications

- Distillation column with pulsation



$$\bar{y} = \Phi(u_1, u_2, \theta_2) \overset{\text{df}}{=} \Phi_1(u_1, \Phi_2(u_2, \theta_2))$$

Two stage identification and it's applications

- Distillation column with pulsation

Measurements: $U_{1N_1n_2} \overset{\text{df}}{=} \left[u_{11n_2} \quad u_{12n_2} \quad \cdots \quad u_{1N_1n_2} \right], \quad Y_{N_1n_2} \overset{\text{df}}{=} \left[y_{1n_2} \quad y_{2n_2} \quad \cdots \quad y_{N_1n_2} \right],$

Performance indices: $Q_{1N_1n_2}(\theta_1) = \frac{1}{N_1} \sum_{n_1=1}^{N_1} q_1\left(y_{n_1n_2}, \Phi_1\left(u_{1n_1n_2}, \theta_1 \right) \right)$

$$\theta^*_{2N_2} = \Psi_{2N_2}\left(U_{2N_2}, \Xi^*_{1N_1N_2} \right) \qquad \theta^*_{1N_1n_2} = \Psi_{1N_1}\left(U_{1N_1n_2}, Y_{N_1n_2} \right)$$

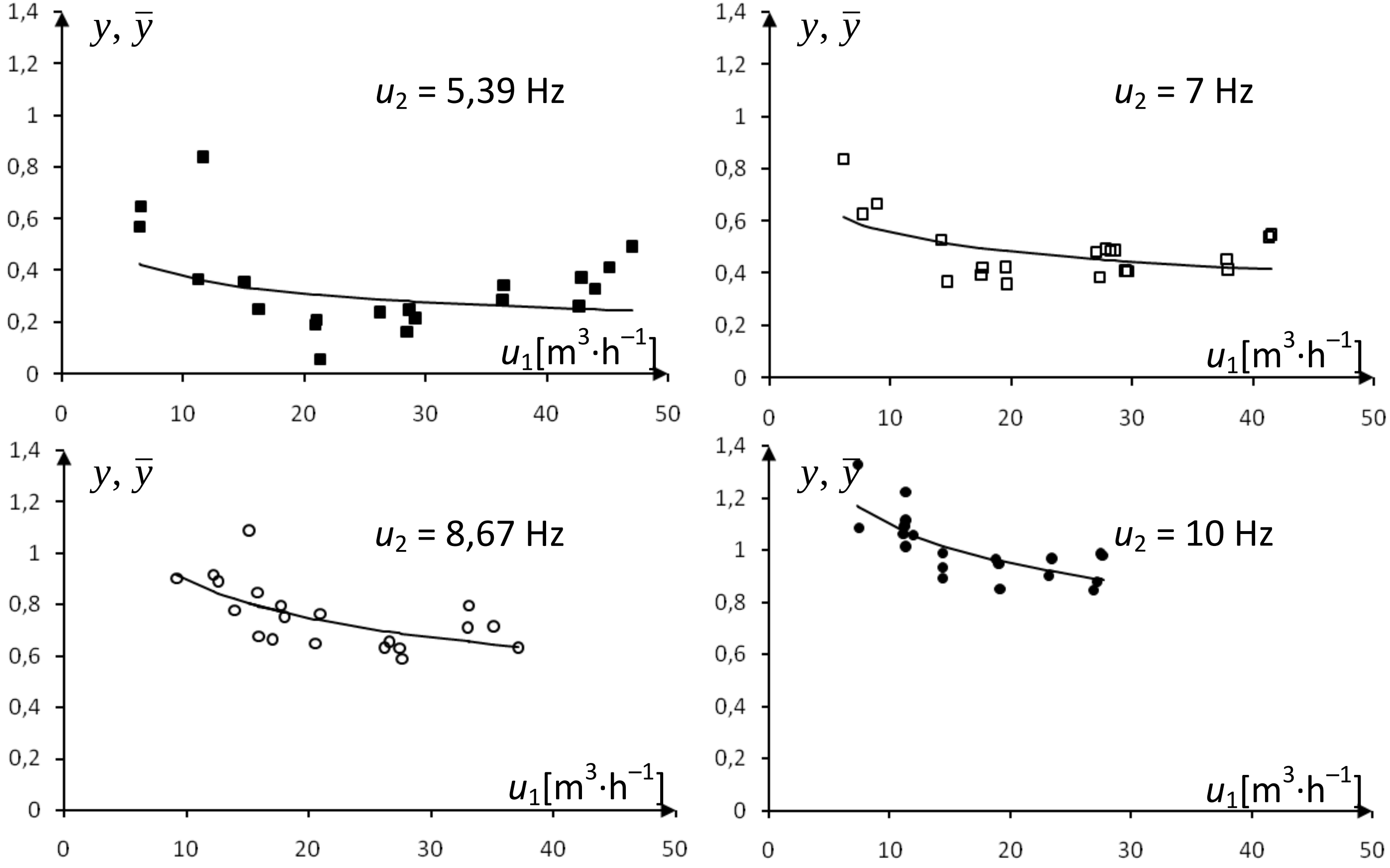
$$\Xi^*_{1N_1N_2} \overset{\text{df}}{=} \left[\theta^*_{1N_11} \quad \theta^*_{1N_12} \quad \cdots \quad \theta^*_{1N_1N_2} \right].$$

$$Q_{2N_2}(\theta_2) = \frac{1}{N_2} \sum_{n_2=1}^{N_2} q_2\left(\theta^*_{1N_1n_2}, \Phi_2\left(u_{2n_2}, \theta_2 \right) \right) \qquad Q_{N_1N_2}(\theta_2) = \frac{1}{N_1N_2} \sum_{n_2=1}^{N_2} \sum_{n_1=1}^{N_1} q_1\left(y_{n_1n_2}, \Phi\left(u_{1n_1n_2}, u_{2n_2}, \theta_2 \right) \right)$$

Two stage identification and it's applications

n_2	$u_{21} = 5{,}39$ (1)		$u_{22} = 7{,}00$ (2)		$u_{23} = 8{,}67$ (3)		$u_{24} = 10{,}00$ (4)	
n_1	$u_{1n_1 1}$	$y_{n_1 1}$	$u_{1n_1 2}$	$y_{n_1 2}$	$u_{1n_1 3}$	$y_{n_1 3}$	$u_{1n_1 4}$	$y_{n_1 4}$
1	6,4	0,572060	6,1	0,838889	9,2	0,903488	7,3	1,3301716
2	6,5	0,648202	7,7	0,628602	12,2	0,916698	7,4	1,0848920
3	11,2	0,366938	8,9	0,666820	12,6	0,891862	11,3	1,0875064
4	11,6	0,840378	14,2	0,529828	13,9	0,780235	11,2	1,0617987
5	15,0	0,357619	14,7	0,369640	15,8	0,849268	11,4	1,2248224
6	16,2	0,252894	17,5	0,393696	15,9	0,676236	11,4	1,0097338
7	20,9	0,191408	17,6	0,423408	17,0	0,665933	11,4	1,1105566
8	21,0	0,211237	19,5	0,424521	17,7	0,798994	11,9	1,0569201
9	21,3	0,057237	19,6	0,359882	18,0	0,753221	14,4	0,9896686
10	26,2	0,240598	27,0	0,484021	15,1	1,089871	14,4	0,8944089
11	28,4	0,162991	27,3	0,386058	20,5	0,651258	14,4	0,9357480
12	28,6	0,249399	27,8	0,493950	20,9	0,764347	18,8	0,9650770
13	29,1	0,217105	28,2	0,487298	26,2	0,634033	19,1	0,9483388
14	36,4	0,343625	28,6	0,490247	26,6	0,657183	19,2	0,8510747
15	36,3	0,290017	29,4	0,411630	27,4	0,630113	23,5	0,9645854
16	42,8	0,373851	29,6	0,408095	27,6	0,588806	23,2	0,9037284
17	42,6	0,263002	37,8	0,453555	33,1	0,796697	26,9	0,8480748
18	43,9	0,331933	37,9	0,416033	33,0	0,712234	27,2	0,8781611
19	45,1	0,414180	41,3	0,539947	35,1	0,716245	27,5	0,9828131
20	47,0	0,494438	41,5	0,549499	37,1	0,633244	27,7	0,9799704

Two stage identification and it's applications



Two stage identification and it's applications

The model: $\bar{y} = \Phi_1(u_1, \theta) = \theta_1^{(2)} u_1^{\theta_1^{(1)}}$

Performance index on the first stage:

$$Q_{1N_1n_2}(\theta_1) = \sum_{n_1=1}^{N_1} \left(\ln y_{n_1n_2} - \ln\left(\theta_1^{(2)} u_{1n_1n_2}{}^{\theta_1^{(1)}} \right) \right)^2 = \sum_{n_1=1}^{N_1} \left(\ln y_{n_1n_2} - \ln \theta_1^{(2)} - \theta_1^{(1)} \ln u_{1n_1n_2} \right)^2.$$

Identification algorithm on the first stage:

$$\theta^*_{1N_1n_2} = \begin{bmatrix} \theta^{*(1)}_{1N_1n_2} \\ \theta^{*(2)}_{1N_1n_2} \end{bmatrix} = \begin{bmatrix} \dfrac{A^{(1)}_{1N_1n_2}}{B_{1N_1n_2}} \\ \exp\left(\dfrac{A^{(2)}_{1N_1n_2}}{B_{1N_1n_2}} \right) \end{bmatrix}$$

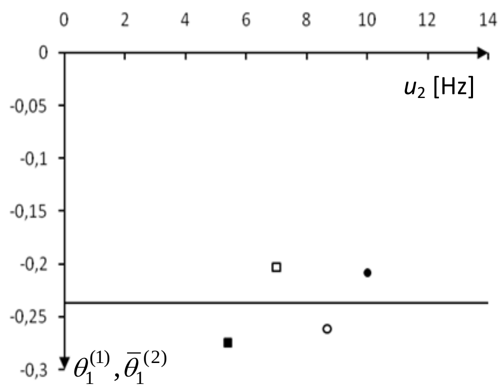
$$A^{(1)}_{1N_1n_2} = \sum_{n_1=1}^{N_1} \ln y_{n_1n_2} \ln u_{1n_1n_2} - \frac{1}{N_1} \left(\sum_{n_1=1}^{N_1} \ln y_{n_1n_2} \right) \left(\sum_{n_1=1}^{N_1} \ln u_{1n_1n_2} \right)$$

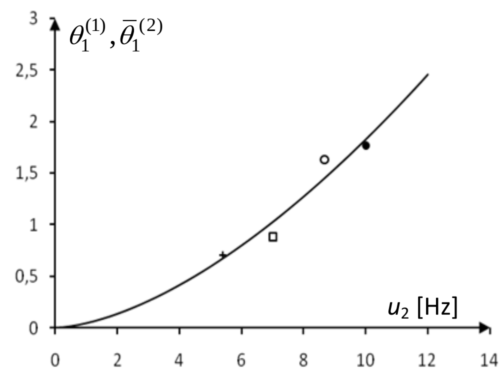
$$A^{(2)}_{N_1n_2} = \frac{1}{N_1} \sum_{n_1=1}^{N_1} \left(\ln u_{1n_1n_2} \right)^2 \sum_{n_1=1}^{N_1} \ln y_{n_1n_2} - \frac{1}{N_1} \left(\sum_{n_1=1}^{N_1} \ln u_{1n_1n_2} \right) \left(\sum_{n_1=1}^{N_1} \ln y_{n_1n_2} \ln u_{1n_1n_2} \right)$$

$$B_{1N_1n_2} = \sum_{n_1=1}^{N_1} \left(\ln u_{1n_1n_2} \right)^2 - \frac{1}{N_1} \left(\sum_{n_1=1}^{N_1} \ln u_{1n_1n_2} \right)^2$$

Two stage identification and it's applications

n_2	1	2	3	4
u_{2n_2}	5,33	7,00	8,67	10,0
$\theta^{*(1)}_{1N_1n_2}$	– 0,274	– 0,203	– 0,260	–0,207
$\theta^{*(2)}_{1N_1n_2}$	0,707	0,886	1,635	1,767





Performance index on the second stage:

$$\bar{\theta}_1 = \Phi_2(u_2, \theta_2) = \begin{bmatrix} \theta_2^{(1)} \\ \theta_2^{(3)} u_2^{\theta_2^{(2)}} \end{bmatrix}$$

$$Q_{2N_2}(\theta_2) = \sum_{n_2=1}^{N_2} \left(\left(\theta^{*(1)}_{1N_1n_2} - \theta_2^{(1)}\right)^2 + \left(\ln \theta^{*(2)}_{1N_1n_2} - \ln\left(\theta_2^{(3)} u_{2n_2}{}^{\theta_2^{(2)}} \right)\right)^2 \right)$$

$$= \sum_{n_2=1}^{N_2} \left(\left(\theta^{*(1)}_{1N_1n_2} - \theta_2^{(1)}\right)^2 + \left(\ln \theta^{*(2)}_{1N_1n_2} - \ln \theta_2^{(3)} - \theta_2^{(2)} \ln u_{2n_2}\right)^2 \right).$$

Two stage identification and it's applications

Identification algorithm on the second stage:

$$\theta^{*}_{2N_2} = \begin{bmatrix} \theta^{*(1)}_{2N_2} \\ \theta^{*(2)}_{2N_2} \\ \theta^{*(3)}_{2N_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_2} \sum_{n_2=1}^{N_2} \theta^{*(1)}_{1N_1 n_2} \\ \dfrac{A^{(1)}_{2N_2}}{B_{2N_2}} \\ \exp\left(\dfrac{A^{(2)}_{2N_2}}{B_{2N_2}} \right) \end{bmatrix}$$

$$A^{(1)}_{2N_2} = \sum_{n_2=1}^{N_2} \ln \theta^{*(2)}_{1n_1 n_2} \ln u_{2n_2} - \frac{1}{N_2} \left(\sum_{n_2=1}^{N_2} \ln \theta^{*(2)}_{1n_1 n_2} \right) \left(\sum_{n_2=1}^{N_2} \ln u_{2n_2} \right)$$

$$A^{(2)}_{2N_2} = \frac{1}{N_2} \left(\sum_{n_2=1}^{N_2} \left(\ln u_{2n_2} \right)^2 \right) \left(\sum_{n_2=1}^{N_2} \ln \theta^{*(2)}_{1n_1 n_2} \right) - \frac{1}{N_2} \left(\sum_{n_2=1}^{N_2} \ln u_{2n_2} \right) \left(\sum_{n_2=1}^{N_2} \ln \theta^{*(2)}_{1n_1 n_2} \ln u_{2n_2} \right)$$

$$B_{2N_2} = \sum_{n_2=1}^{N_2} \left(\ln u_{2n_2} \right)^2 - \frac{1}{N_2} \left(\sum_{n_2=1}^{N_2} \ln u_{2n_2} \right)^2$$

Two stage identification and it's applications

- Direct approach

The model:

$$\bar{y} = \Phi(u_1, u_2, \theta_2) = \theta_2^{(3)} u_2^{\theta_2^{(2)}} u_1^{\theta_2^{(1)}},$$

Performance index:

$$Q_{N_1 N_2}(\theta_2) = \sum_{n_2=1}^{N_2} \sum_{n_1=1}^{N_1} \left(\ln y_{n_1 n_2} - \ln\left(\theta_2^{(3)} u_{2n_2}^{\theta_2^{(2)}} u_{1n_1 n_2}^{\theta_2^{(1)}} \right) \right)^2$$

$$= \sum_{n_2=1}^{N_2} \sum_{n_1=1}^{N_1} \left(\ln y_{n_1 n_2} - \ln \theta_2^{(3)} - \theta_2^{(2)} \ln u_{2n_2} - \theta_2^{(1)} \ln u_{1n_1 n_2} \right)^2$$

Two stage identification and it's applications

- Direct approach

Identification algorithm: $$\tilde{\theta}^{*}_{2N_1N_2} = \begin{bmatrix} \tilde{\theta}^{*(1)}_{2N_1N_2} \\ \tilde{\theta}^{*(2)}_{2N_1N_2} \\ \tilde{\theta}^{*(3)}_{2N_1N_2} \end{bmatrix} = \begin{bmatrix} A^{(1)}_{N_1N_2} \\ A^{(2)}_{N_1N_2} \\ \exp A^{(3)}_{N_1N_2} \end{bmatrix}$$

$$A_{N_1N_2} \overset{\text{df}}{=} \begin{bmatrix} A^{(1)}_{N_1N_2} \\ A^{(2)}_{N_1N_2} \\ A^{(3)}_{N_1N_2} \end{bmatrix} = M^{-1}_{N_1N_2} b_{N_1N_2}$$

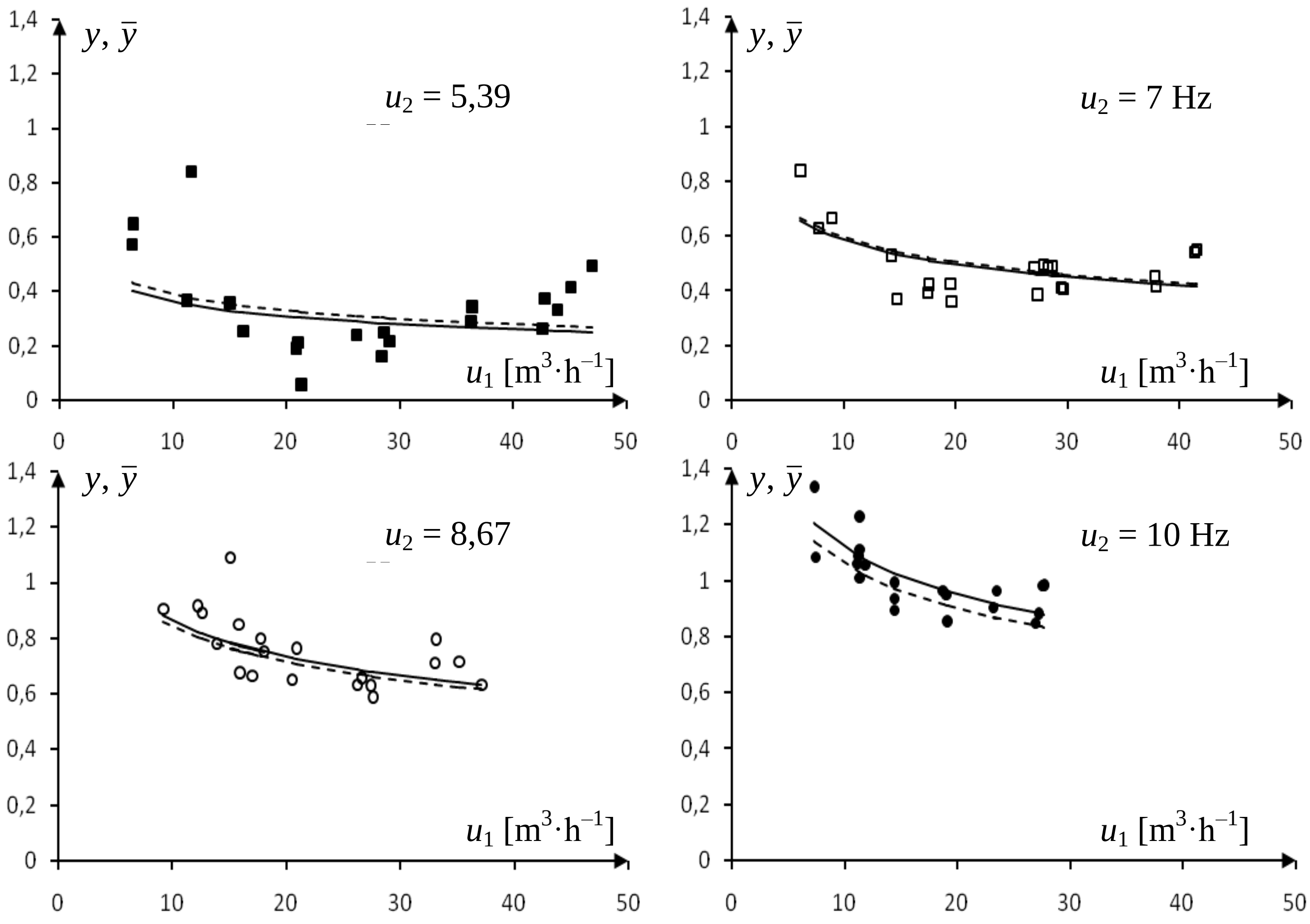
$$M_{N_1N_2} = \sum_{n_2=1}^{N_2} \sum_{n_1=1}^{N_1} \begin{bmatrix} \ln u_{1n_1n_2} \\ \ln u_{2n_2} \\ 1 \end{bmatrix} \begin{bmatrix} \ln u_{1n_1n_2} & \ln u_{2n_2} & 1 \end{bmatrix} \qquad b_{N_1N_2} = \sum_{n_2=1}^{N_2} \sum_{n_1=1}^{N_1} \begin{bmatrix} \ln u_{1n_1n_2} \\ \ln u_{2n_2} \\ 1 \end{bmatrix} \ln y_{n_1n_2}$$

Two stage identification and it's applications

- Direct approach

Approach	θ_2	$\theta_2^{(1)}$	$\theta_2^{(2)}$	$\theta_2^{(3)}$	$Q_{N_1 N_2}(\theta_2)$
Two-stage	$\theta_2 = \theta^*_{2N_2}$	–0,236	1,624	0,043	1,053014
Direct	$\theta_2 = \tilde{\theta}^*_{2N_1N_2}$	–0,237	1,826	0,029	1,016943

Two stage identification and it's applications



Egzamin

- Identyfikacja systemów
- Termin: 0 – ostatni wykład
  20.06.2017 (środa) sala 29, budynek D-1, godz. 7^{30} - 9^{00}
- Warunki terminu 0:
  - Aktualna (tegoroczna) ocena z formy pomocniczej ≥ 3.5
  - Obecność na ostatnim wykładzie
  - Nieobecność jest równoważna z rezygnacją ze zwolnienia
- Termin 1:
  28.06.2017 (środa) sala 409 budynek B-4 godz. 7^{30} - 9^{00}
- Termin 2:
  5.07.2017 (środa) sala 409 budynek B-4 godz. 7^{30} - 9^{00}

Thank you for attention