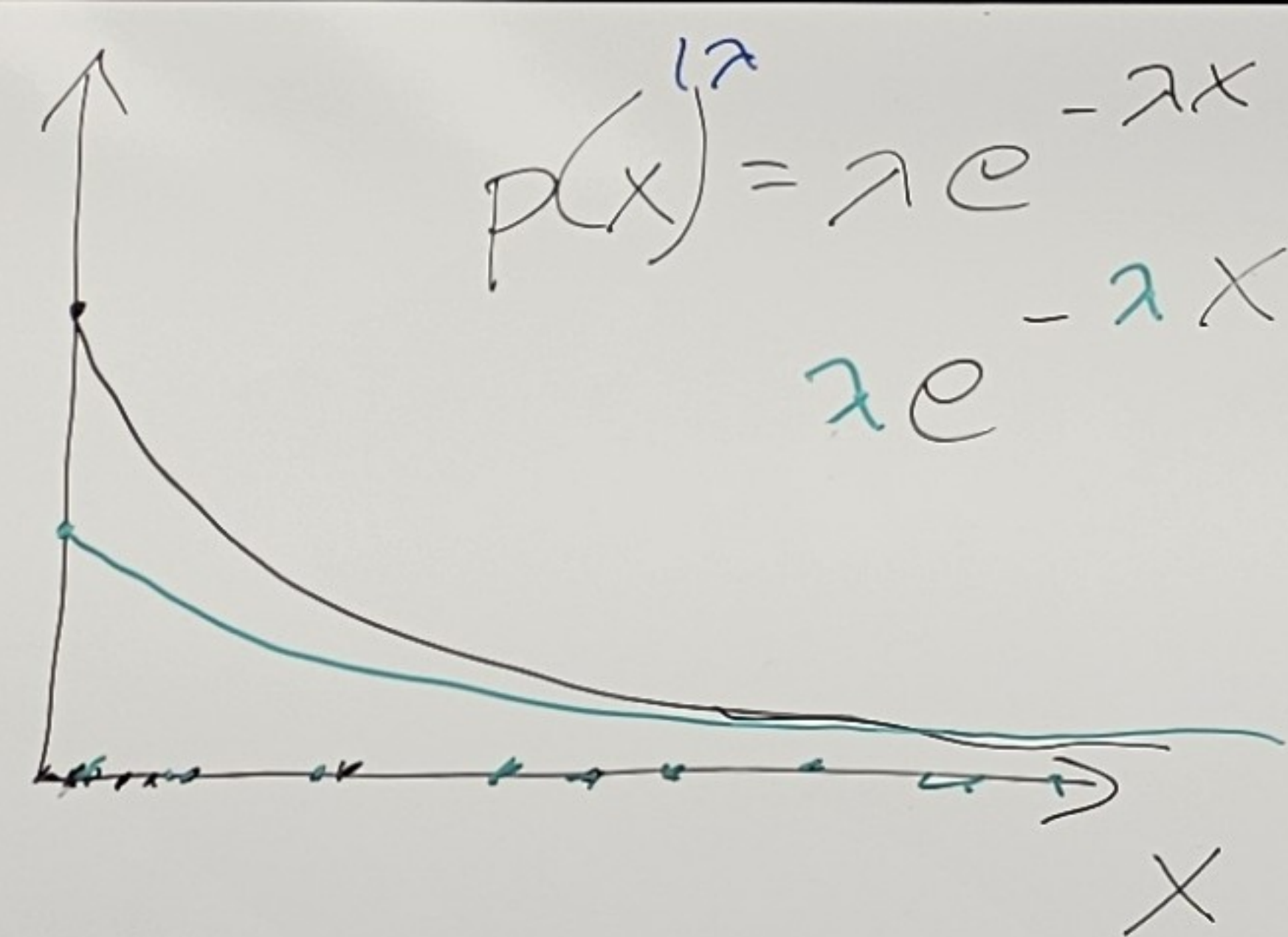




$$L(\lambda) = \prod_{i=1}^N p(x_i | \lambda)$$

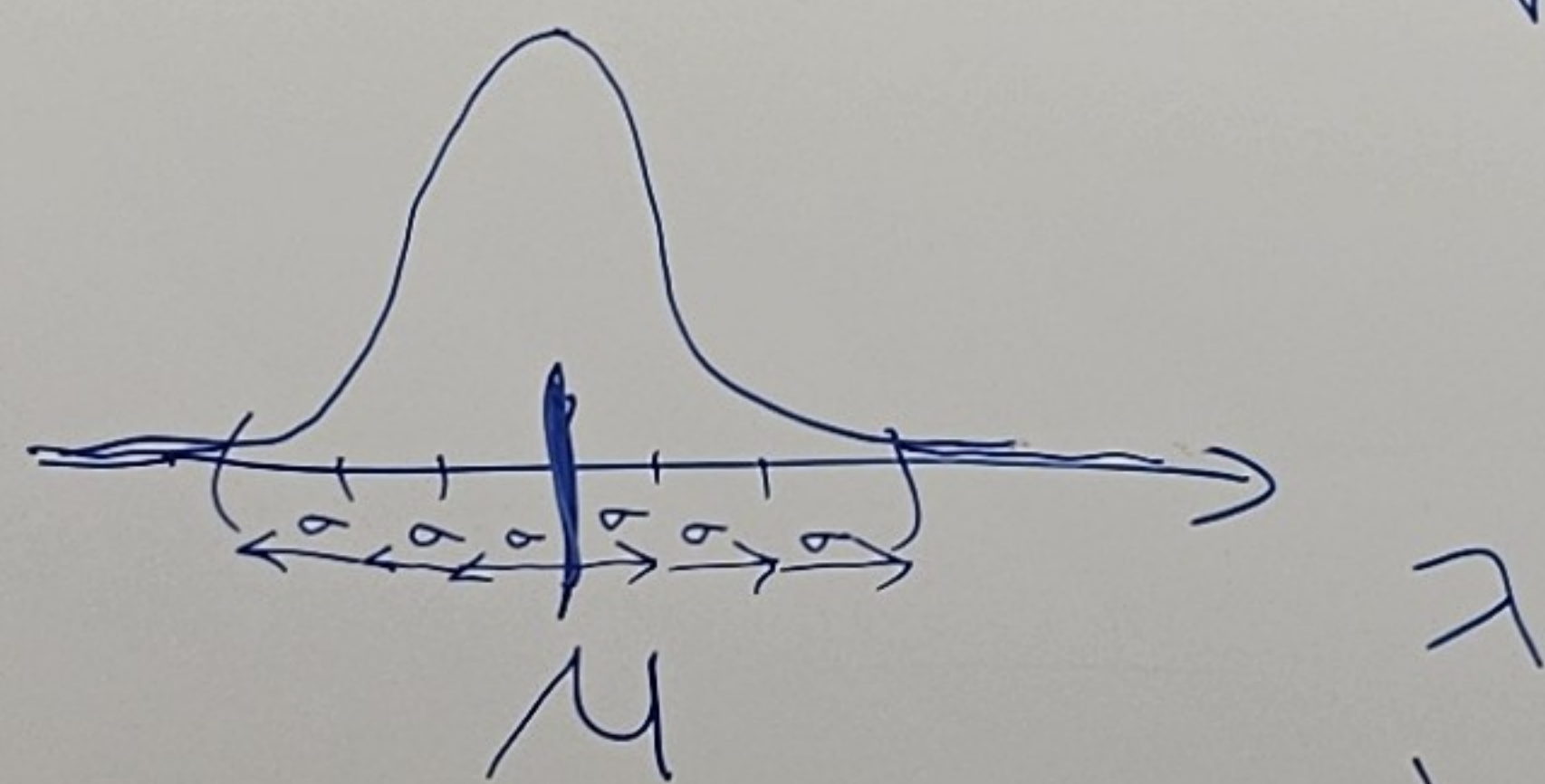
$$\frac{d \log L}{d \lambda} = 0$$

$$\lambda = \frac{1}{\bar{x}}$$

Przekonanie

$$p(\lambda | \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\lambda - \mu}{\sigma} \right)^2 \right]$$

$\mu, \sigma = \text{const.}$



$$\lambda^2 + \lambda (N \bar{x} \sigma^2 - \mu) - N \sigma^2 = 0$$

$$\frac{d \log \text{MAP}(\lambda)}{d \lambda} \rightarrow 0$$

$$\text{MAP}(\lambda) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\lambda - \mu}{\sigma} \right)^2 \right] \prod_{i=1}^N \lambda \exp[-\lambda x_i]$$

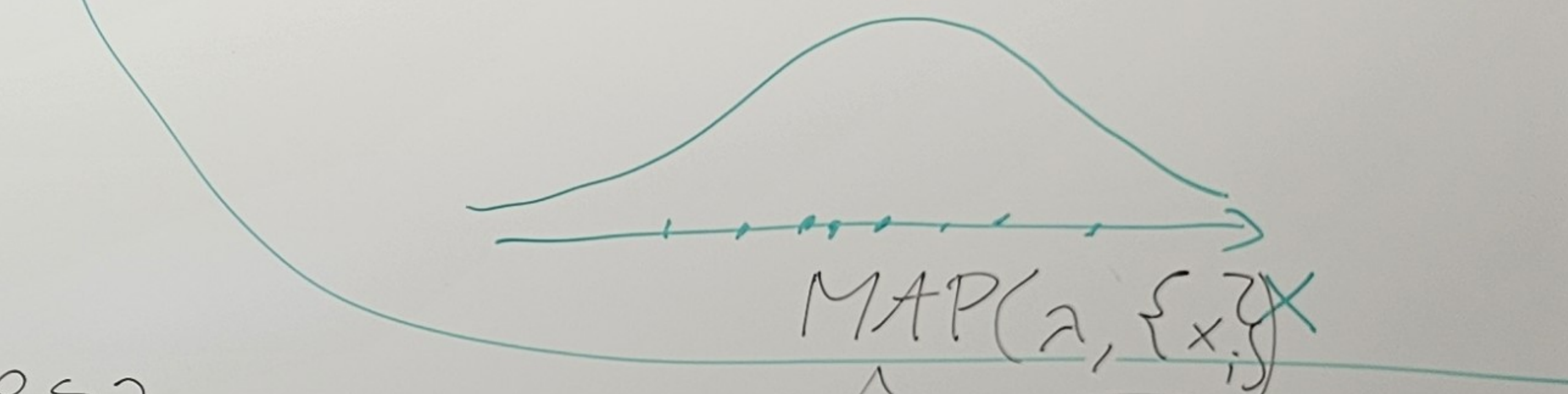
Met. Bayesa

$$\max_{\lambda} p(\lambda | x) = \max_{\lambda} \underbrace{p(x | \lambda) \cdot p(\lambda)}_{\text{MAP}(\lambda, \{x_i\})}$$

$$\max_{\lambda} \text{MAP}(\lambda, \{x_i\})$$

u domu

$$p(x | \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2 \right]$$



$$\frac{N}{\lambda} = N\bar{x} \quad \lambda = \frac{1}{\bar{x}}$$

$$\lambda = \frac{\mu - N\sigma^2\bar{x} + \sigma^2}{2} =$$

$$\lambda =$$

$$\lambda = \hat{\lambda} = \frac{\mu - \sigma^2(N\bar{x} - 1)}{2}$$

$$\log \text{MAP}(\lambda) = -\log \sigma \sqrt{2\pi} - \frac{1}{2} \left(\frac{\lambda - \mu}{\sigma} \right)^2 + \sum_{i=1}^N \log \lambda - \lambda x_i =$$

$$\mu = \bar{x}$$

$$= -\log \sigma \sqrt{2\pi} - \frac{1}{2\sigma^2} (\lambda - \mu)^2 + N \log \lambda - \lambda N \bar{x}$$

$$\frac{d}{d\lambda} = -\frac{1}{\sigma^2} (\lambda - \mu) + N \cdot \frac{1}{\lambda} - N\bar{x} = 0 : \mu - \lambda + \frac{N\sigma^2}{\lambda} - N\sigma^2\bar{x} = 0$$

$$\mu\lambda - \lambda^2 + N\sigma^2 - \lambda N\sigma^2\bar{x} = 0$$

$$\lambda^2 + \lambda(N\sigma^2\bar{x} - \mu) - N\sigma^2 = 0$$

$$\Delta = (N\sigma^2\bar{x} - \mu)^2 + 4N\sigma^2$$

$$\lambda = \frac{\mu - N\sigma^2\bar{x} \pm \sqrt{\Delta}}{2}$$

$$\Delta = N^2\sigma^4\bar{x}^2 - 2N\sigma^2\bar{x}\mu + \mu^2 + 4N\sigma^2$$

$$\text{dla } \sigma = 0$$

$$\Delta = \mu^2$$

$$\lambda = \frac{\mu - \mu}{2} = 0$$

$$\lambda = \mu$$

$$\text{dla } \sigma \rightarrow \infty$$

$$\sqrt{\Delta} \rightarrow \sigma^2$$