$$\mu = \begin{bmatrix} \mu^{(1)} \\ \vdots \\ \mu^{(K)} \end{bmatrix}$$

$$\det(I) = 1$$

$$\det(AA^{-1}) = 1$$

$$\det(A)\det(A^{-1}) = 1$$

$$\det A^{-1} = \frac{1}{\det A}$$

$$\frac{\partial L}{\partial \mu} = \sum_{i=1}^{N} H^{-1}(x_i - \mu) = 0_K$$

$$H^{-1}\sum_{i=1}^{N}(x_i - \mu) = 0$$

$$p(x_i;\mu,H) = \frac{1}{(2\pi)^{K/2}(\det H)^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(x-\mu)^T H^{-1}(x-\mu)\right]$$

$$L(\mu,H) = \prod_{i=1}^{N} p(x_i;\mu,H) = \sum_{i=1}^{N} \log p(x_i;\mu,H)$$

$$\log p(x_i;\mu,H) = -\underbrace{\frac{K}{2}\log 2\pi}_{\alpha} - \frac{1}{2}\log\det H - \frac{1}{2}\left(x^T H^{-1} x - x^T H^{-1}\mu - \mu^T H^{-1} x + \mu^T H^{-1}\mu\right)$$

$$= -\alpha + \frac{1}{2}\log(\det H)^{-1} - \frac{1}{2}(- \cdots -)$$

$$= -\alpha + \frac{1}{2}\log\det H^{-1} - \frac{1}{2}(- \cdots -)$$

$$H^T = H$$

$$(H^{-1})^T$$

$$\frac{\partial \log p(x_i;\mu,H)}{\partial \mu} = -\frac{1}{2}\left(0 - 2H^{-1}x + 2H^{-1}\mu\right)$$

$$\frac{\partial \log p(x_i;\mu,H)}{\partial H^{-1}} = \frac{1}{2}H - \frac{1}{2}\underbrace{\left(xx^T - x\mu^T - \mu x^T + \mu\mu^T\right)}_{=(x-\mu)(x-\mu)^T}$$

$$\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma^2}\right)^2\right]$$

$$\frac{\partial}{\partial w} w^T A w = (A + A^T)w$$

$$\frac{\partial}{\partial A} b^T A x = b x^T$$

$$\frac{\partial}{\partial A}\log\det A = (A^{-1})^T$$

$$\frac{\partial}{\partial w} w^T x = \frac{\partial}{\partial w} x^T w = x$$

$$\frac{\partial}{\partial w} w^T w = 2w$$

$$\det A^{-1} = \frac{1}{\det(A)}$$

$$A^2 = A$$

$$\frac{\partial L}{\partial H^{-1}} = \sum_{i=1}^{N} (\;\;) = 0_{K\times K}$$

$$= NH - \sum_{i=1}^{N} (x_i - \mu)(x_i - \mu)^T$$

$$\boxed{\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} x_i = \hat{X}}$$

$$\sum_{i=0}^{N} (x_i - \mu) = 0$$

$$H^{-1} \sum_{i=1}^{N} (x_i - \mu) = 0$$

$$\boxed{\hat{H} = \frac{1}{N}\sum_{i=1}^{N} (x_i - \mu)(x_i - \mu)^T}$$

$m = 0_{1\times N}$

$$p(X_i Y | \cancel{m}, R) = \frac{1}{(2\pi)^{\frac{N}{2}} (\det R)^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(Y - \Theta^T X) R^{-1} (Y - \Theta^T X)^T\right]$$

I poziom

W domu: wylicz $\hat{R}_{ML}$

+II poziom

$$p(\Theta) = \frac{1}{(2\pi)^{\frac{K}{2}} (\det H)^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(\Theta - \mu)^T H^{-1} (\Theta - \mu)\right]$$

μ

H

$$\frac{\partial \log p(x_i \mu, H)}{\partial H^{-1}} = \frac{1}{2} H - \frac{1}{2}(xx^T - x\mu^T - \mu x^T + \mu\mu^T) = (x-\mu)(x-\mu)^T$$

Dla modelu $\bar{Y} = \Theta^T X$

$$Y = \Theta^T X + Z$$

$$\underset{1\times N}{Z} = \underset{1\times N}{Y} - \Theta^T \underset{K\times N}{X}$$

$$Y - \bar{Y}$$

$$\frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma^2}\right)^2\right]$$

$$\frac{\partial}{\partial w} w^T A w = (A + A^T) w$$

$$\frac{\partial}{\partial A} b^T A x = b x^T$$

$$\frac{\partial}{\partial A} \log \det A = (A^{-1})^T$$

$$\frac{\partial}{\partial w} w^T x = \frac{\partial}{\partial w} x^T w = x$$

$$\frac{\partial}{\partial w} w^T w = 2w$$

$$\det A^{-1} = \frac{1}{\det(A)}$$

$$A^2 = A$$

$$\frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

$$x = \begin{bmatrix} x^{(1)} \\ \vdots \\ x^{(K)} \end{bmatrix} \quad \mu = \begin{bmatrix} \mu^{(1)} \\ \vdots \\ \mu^{(K)} \end{bmatrix}$$

$$\det(I) = 1$$

$$\det(AA^{-1}) = 1$$

$$\{x_i\}_{i=1}^{N}$$

$$\det(A)\cdot\det(A^{-1}) = 1$$

$$\det A^{-1} = \frac{1}{\det A}$$

$$H^T = H \qquad (H^{-1})^T = (H^T)^{-1}$$

$$p(x|\mu, H) = \frac{1}{(2\pi)^{K/2}(\det H)^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(x-\mu)^T H^{-1}(x-\mu)\right]$$

$$\log p(x|\mu,H) = -\frac{K}{2}\log 2\pi - \frac{1}{2}\log\det H - \frac{1}{2}\left(x^T H^{-1} x - x^T H^{-1}\mu - \mu^T H^{-1} x + \mu^T H^{-1}\mu\right)$$

$$\frac{\partial \log p}{\partial \mu} = -\frac{1}{2}\left(-2H^{-1}x + 2H^{-1}\mu\right) = H^{-1}(x-\mu)$$

$$\frac{\partial \log p}{\partial H^{-1}} = \frac{1}{2}H - \frac{1}{2}\Big(\underbrace{xx^T - x\mu^T - \mu x^T + \mu\mu^T}_{(x-\mu)(x-\mu)^T}\Big)$$

$$\frac{\partial}{\partial w} w^T x = \frac{\partial}{\partial w} x^T w = x$$

$$\frac{\partial}{\partial w} w^T w = 2w$$

$$\frac{\partial}{\partial w} w^T A w = (A + A^T)w$$

$$\frac{\partial}{\partial A} b^T A x = b x^T$$

$$\frac{\partial}{\partial A} \log\det A = (A^{-1})^T$$

$$\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

$$x=\begin{bmatrix}x^{(1)}\\ \vdots\\ x^{(k)}\end{bmatrix}\quad \mu=\begin{bmatrix}\mu^{(1)}\\ \vdots\\ \mu^{(k)}\end{bmatrix}$$

$$\det(I)=1$$

$$\det(AA^{-1})=1$$

$$\{x_i\}_{i=1}^{N}$$

$$\det(A)\cdot\det(A^{-1})=1$$

$$\det A^{-1}=\frac{1}{\det A}$$

$$H^T=H \qquad (H^{-1})^T=(H^T)^{-1}$$

$$H^{-1}\sum_{i=1}^{N}(x_i-\mu)=0 \Rightarrow \boxed{\mu=\frac{1}{N}\sum_{i=1}^{N}x_i}$$

$$\boxed{H=\frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)(x_i-\mu)^T}$$

$$L(\mu,H)=\prod_{i=1}^{N}p(x_i\mid\mu,H)$$

$$\frac{\partial\log p}{\partial\mu}=-\frac{1}{2}\left(-2H^{-1}x+2H^{-1}\mu\right)=H^{-1}(x-\mu)$$

$$\frac{\partial\log p}{\partial H^{-1}}=\frac{1}{2}H-\frac{1}{2}\left(xx^T-x\mu^T-\mu x^T+\mu\mu^T\right)$$

$$(x-\mu)(x-\mu)^T$$

$$\frac{\partial}{\partial w}w^Tx=\frac{\partial}{\partial w}x^Tw=x$$

$$\frac{\partial}{\partial w}w^Tw=2w$$

$$\frac{\partial}{\partial w}w^TAw=(A+A^T)w$$

$$\frac{\partial}{\partial A}b^TAx=bx^T$$

$$\frac{\partial}{\partial A}\log\det A=(A^{-1})^T$$

$$p(\text{Dane}\mid\text{Model})$$

$$\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

$$x=\begin{bmatrix}x^{(1)}\\ \vdots \\ x^{(K)}\end{bmatrix}\quad \mu=\begin{bmatrix}\mu^{(1)}\\ \vdots \\ \mu^{(K)}\end{bmatrix}$$

$\det(I)=1$

$\det(AA^{-1})=1$

$\{x_i\}$,

$\det(A)\cdot\det(A^{-1})=1$

$$\det A^{-1}=\frac{1}{\det A}$$

$H^T=H \qquad (H^{-1})^T=(H^T)^{-1}$

$X,\ Y_{1 \times N}$

$Y=\Theta^T X$

$Y=\Theta^T X+Z$

$L(Z)=L(Y-\Theta^T X)=$

$=L(Y,X|\Theta)$

$$p(Z)=\frac{1}{(2\pi)^{N/2}(\det R)^{\frac{1}{2}}}\exp\left[-\frac{1}{2}(Y-\Theta^T X)R^{-1}(Y-\Theta^T X)^T\right]$$

$R_{N\times N}$

w domu: $\hat{\Theta}_{ML}=?$

$2+3=5$

$$\frac{\partial}{\partial w}w^Tx=\frac{\partial}{\partial w}x^Tw=x$$

$$\frac{\partial}{\partial w}w^Tw=2w$$

$$\frac{\partial}{\partial w}w^TAw=(A+A^T)w$$

$$\frac{\partial}{\partial A}b^TAx=bx^T$$

$$\frac{\partial}{\partial A}\log\det A=(A^{-1})^T$$

$p(\text{Dane}\,|\,\text{Model})$

$$p(\{x_i\}_{i=1}^N \mid \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

$$\{x_i\}_{i=1}^N$$

$$\mu = \begin{bmatrix} \mu^{(1)} \\ \vdots \\ \mu^{(K)} \end{bmatrix} \quad x_i = \begin{bmatrix} x_i^{(1)} \\ \vdots \\ x_i^{(K)} \end{bmatrix}$$

$$\det(I) = 1$$

$$\det(AA^{-1}) = 1$$

$$\det(A) \cdot \det(A^{-1}) = 1$$

$$\det(A^{-1}) = \frac{1}{\det A}$$

$$H^T = H$$

$$(H^{-1})^T = (H^T)^{-1}$$

$$p(x \mid \mu, H) = \frac{1}{(2\pi)^{K/2} (\det H)^{1/2}} \exp\left[-\frac{1}{2}(x-\mu)^T H^{-1} (x-\mu)\right]$$

log

$$L(\mu, H) = \sum_{i=1}^N \log p(x_i \mid \mu, H)$$

$$\frac{1}{2}\log(\det H)^{-1} \quad \frac{1}{2}\log\frac{1}{\det H} \quad \frac{1}{2}\log\det(H^{-1})$$

$$\log p(x \mid \mu, H) = -\frac{K}{2}\log 2\pi - \frac{1}{2}\log\det H - \frac{1}{2}\left(x^T H^{-1} x - x^T H^{-1}\mu - \mu^T H^{-1} x + \mu^T H^{-1}\mu\right)$$

$$\frac{\partial \log p}{\partial \mu} = -\frac{1}{2}\left(-2H^{-1}x - 2H^{-1}\mu\right) = H^{-1}(x-\mu)$$

$$\frac{\partial L}{\partial \mu} = 0$$

$$H^{-1}\sum_{i=1}^N (x_i - \mu) = 0_K$$

$$\mu^* = \frac{1}{N}\sum_{i=1}^N x_i$$

$$\frac{\partial \log p}{\partial H^{-1}} = \frac{1}{2}H - \frac{1}{2}\left(xx^T - x\mu^T - \mu x^T + \mu\mu^T\right)$$

$$(x-\mu)(x-\mu)^T$$

$$\frac{\partial L}{\partial H^{-1}} = NH - \frac{1}{N}\sum_{i=1}^N (x_i - \mu)(x_i - \mu)^T$$

$$H = \frac{1}{N}\sum_{i=1}^N (x_i - \mu)(x_i - \mu)^T$$

$$\frac{\partial}{\partial w} w^T x = \frac{\partial}{\partial w} x^T w = x$$

$$\frac{\partial}{\partial w} w^T w = 2w$$

$$\frac{\partial}{\partial w} w^T A w = (A + A^T) w$$

$$\frac{\partial}{\partial A} b^T A x = b x^T$$

$$\frac{\partial}{\partial A} \log\det A = (A^{-1})^T$$

$$p(\underbrace{\text{Dane}}_{\{x_i\}_{i=1}^N} \mid \underbrace{\text{Model}}_{\mu, H})$$

$\underline{X}, \underline{Y}_{1\times N}$

$\underline{Y} = \Theta^T \underline{X}_{K\times N}$

$\underline{Y} = \Theta^T \underline{X} + \underline{Z}_{1\times N}$

$$p(\underline{Z}) = \frac{1}{(2\pi)^{N/2} (\det R)^{1/2}} \exp\left[-\frac{1}{2}(\underline{Y}-\Theta^T\underline{X}) R^{-1} (\underline{Y}-\Theta^T\underline{X})^T\right]$$

R. znane

$L(\Theta) =$

w domu: $\hat{\Theta} = \arg\max L(\Theta)$

$$\frac{\partial L}{\partial \Theta} \rightarrow \frac{\partial \log L}{\partial \Theta} = 0$$

$\underline{X}, \underline{Y}$

p(Dane | Model)