fun. wiaryg. ↓

$$P(x \mid \Theta) = \Theta^{x}(1-\Theta)^{N-x}$$

1° $X = [O\ O\ O\ R]$ $\quad \Theta = \frac{1}{2} \quad P(X \mid \Theta = \frac{1}{2}) = 1/16 = 0{,}0625$

2° $[O\ O\ O\ R] = \Theta = \frac{2}{3} \quad P(X \mid \Theta = \frac{2}{3}) = \frac{8}{81} \approx 0{,}0988$

$\Theta = \frac{3}{4} \quad P(X \mid \Theta = \frac{3}{4}) = \frac{27}{256} \approx 0{,}105$

N rzutów monetą $\qquad$ x razy wypadł orzeł

$\Theta = ?$ (prawd. że wypadnie O)

max. f. wiar.:

$$\frac{x}{\Theta} = \frac{N-x}{1-\Theta}$$

$$x(1-\Theta) - (N-x)\Theta = 0$$

$$x - \Theta x - \Theta N + \cancel{x\Theta} = 0$$

$$x - \Theta N = 0$$

$$\boxed{\hat{\Theta} = \frac{x}{N}}$$

$$\ln\left(P(x \mid \Theta) = \Theta^{x}(1-\Theta)^{N-x}\right)$$

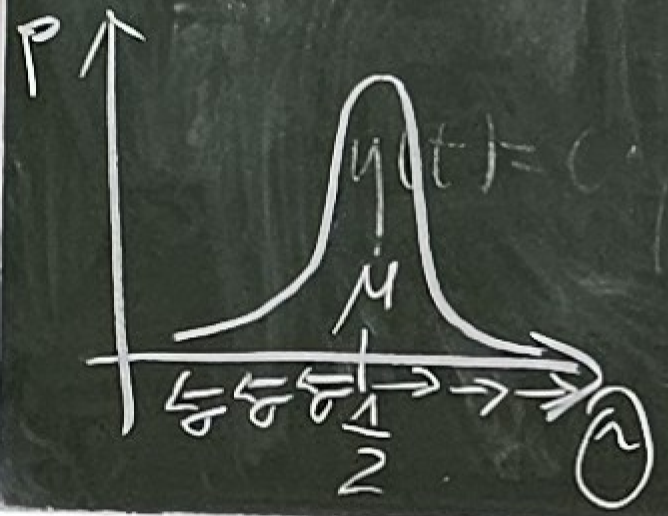
$$\ln(P(x \mid \Theta)) = \ln(\Theta^{x}) + \ln((1-\Theta)^{N-x})$$

$$\ln(P(x \mid \Theta)) = x \ln\Theta + (N-x)\ln(1-\Theta)$$

$$\frac{d}{d\Theta} = \frac{x}{\Theta} + (N-x)\cdot\frac{-1}{1-\Theta} =$$

$$= \frac{x}{\Theta} - \frac{N-x}{1-\Theta} = 0$$

$\mu, \sigma = \text{const.}$



$$p(\Theta) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\Theta-\mu}{\sigma}\right)^{2}\right]$$

$$\frac{x}{\theta} + \frac{n-x}{1-\theta} = \frac{\theta-\mu}{\sigma^2}$$

$$\frac{x - x\theta + n\theta - x\theta}{\theta(1-\theta)} = \frac{\theta-\mu}{\sigma^2}$$

$$\frac{x - 2x\theta + n\theta}{\theta(1-\theta)} = \frac{\theta-\mu}{\sigma^2}$$

$$(x - 2x\theta + n\theta)\sigma^2 = \theta(1-\theta)(\theta-\mu)$$

$$x\sigma^2 - 2x\theta\sigma^2 + n\sigma^2\theta = (\theta-\theta^2)(\theta-\mu)$$

$$x\sigma^2 - 2x\theta\sigma^2 + n\sigma^2\theta = \theta^2 - \mu\theta - \theta^3 + \mu\theta^2$$

$$x\sigma^2 = -\theta^3 + \theta^2(\mu-1) + \theta(2x\sigma^2 - n\sigma^2)$$

$$=$$

$$p(\theta|x) \sim p(x|\theta)\cdot p(\theta)$$

$$\ln p(\theta|x) = \ln\left[\theta^x (1-\theta)^{n-x} \cdot \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\theta-\mu}{\sigma}\right)^2\right]\right] =$$

$$= \ln\left(\theta^x (1-\theta)^{n-x}\right) + \ln\left(\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\theta-\mu}{\sigma}\right)^2\right)\right) =$$

$$x\ln\theta + (n-x)\ln(1-\theta) + \ln\frac{1}{\sigma\sqrt{2\pi}} + \ln e^{-\frac{1}{2}\left(\frac{\theta-\mu}{\sigma}\right)^2}$$

$$=$$

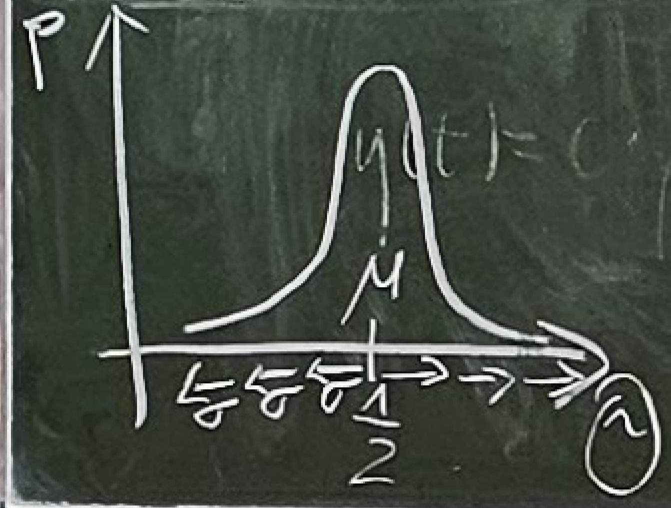
$$x\ln\theta + (n-x)\ln(1-\theta) + \ln\frac{1}{\sigma\sqrt{2\pi}} +$$

$$-\frac{1}{2}\left(\frac{\theta-\mu}{\sigma}\right)^2$$

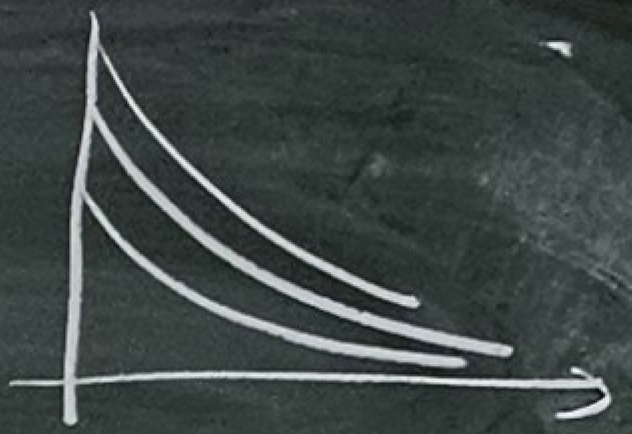
$$\frac{x}{\theta} + \frac{n-x}{1-\theta} - \frac{\theta-\mu}{\sigma^2} = 0$$

$$\frac{x}{\theta} + \frac{n-x}{1-\theta} = \frac{\theta-\mu}{\sigma^2}$$

$\mu, \sigma = \text{const.}$



$$p(\theta) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\theta-\mu}{\sigma}\right)^2\right]$$



$$\frac{N}{\lambda} - \sum_{i=1}^{N} x_i = 0$$

$$\frac{N}{\lambda} = \sum_{i=1}^{N} x_i$$

$$\lambda = \frac{N}{\sum_{i=1}^{N} x_i} = \frac{1}{\hat{X}}$$

$$p(x|\lambda) = \lambda \exp[-\lambda x]$$

$$\underline{X} = [x_1 \ x_2 \ \dots \ x_N]$$

$$\Theta \equiv \lambda$$

x_i – niezalezne

$$L(\Theta) \equiv p(\underline{X}|\Theta) = \prod_{i=1}^{N} p(x_i|\lambda)$$

$$\ln L = \sum_{i=1}^{N} \ln(p(x_i|\lambda))$$

$$\sum_{i=1}^{N} \ln(\lambda e^{-\lambda x_i}) = \sum_{i=1}^{N} \ln\lambda + \sum_{i=1}^{N} \ln e^{-\lambda x_i} = \ln\lambda \sum_{i=1}^{N} 1 - \lambda$$

$$\overbrace{N \cdot \frac{1}{N}\sum_{i=1}^{N} x_i}^{\bar{x}}$$

$$\sum_{i=1}^{N} x_i = N \ln\lambda - \lambda \sum_{i=1}^{N} x_i$$

W domu Weibulla

r-d Normalny

N rzutów monetą

cieszy nas R

X - liczba sukcesów

Θ - P(R)

$$P(x|\Theta) = \Theta^x (1-\Theta)^{N-x}$$

|||

$L(\Theta)$

$$\frac{\partial}{\partial \Theta} \Theta^x (1-\Theta)^{N-x} =$$

$$\ln \Theta^x (1-\Theta)^{N-x} = \ln \Theta^x + \ln (1-\Theta)^{N-x} = \ln \Theta^x + (N-x) \ln(1-\Theta) =$$

$$= x \ln \Theta + (N-x) \ln(1-\Theta)$$

$$\frac{\partial}{\partial \Theta} x \ln \Theta + (N-x) \ln(1-\Theta) = \frac{x}{\Theta} - \frac{N-x}{1-\Theta}$$

$$\frac{x}{\Theta} + \frac{N-x}{1-\Theta} = 0 \qquad \frac{(1-\Theta)x - (N-x)\Theta}{(1-\Theta)\Theta} = 0 \qquad \Theta \neq 0 \quad 1-\Theta \neq 0$$

$$(1-\Theta)x - (N-x)\Theta = 0$$

$$x - \cancel{\Theta x} - N\Theta + \cancel{\Theta x} = 0$$

$$\Theta = \frac{x}{N}$$

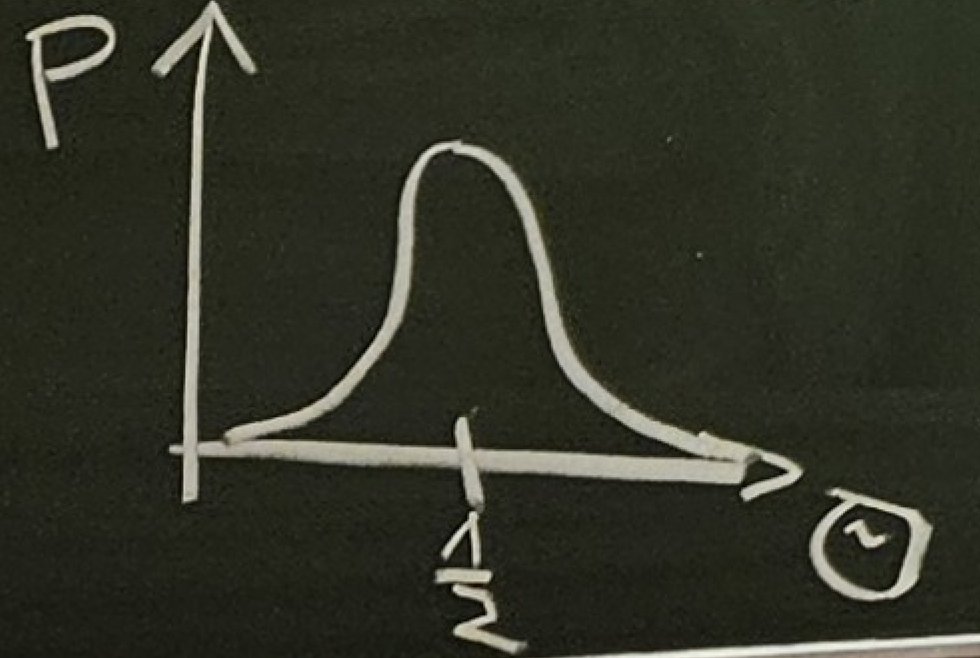
N rzutów monetą
cieszy nas R
X - liczba sukcesów

Θ - P(R)   ln ln

$$P(x|\Theta) = \Theta^x (1-\Theta)^{N-x}$$

|||

L(Θ)



$$p(\Theta) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\Theta-\mu}{\sigma}\right)^2\right]$$

$\mu, \sigma = \text{const.}$

$$p(\Theta|x) = \overbrace{p(x|\Theta)}^{L(\Theta)} \cdot p(\Theta)$$

$$\log p(\Theta|x) = \log p(x|\Theta) + \log p(\Theta)$$

$$\ln p(x|\Theta) = \ln \Theta^x (1-\Theta)^{N-x} = x \ln\Theta + (N-x)\ln(1-\Theta)$$

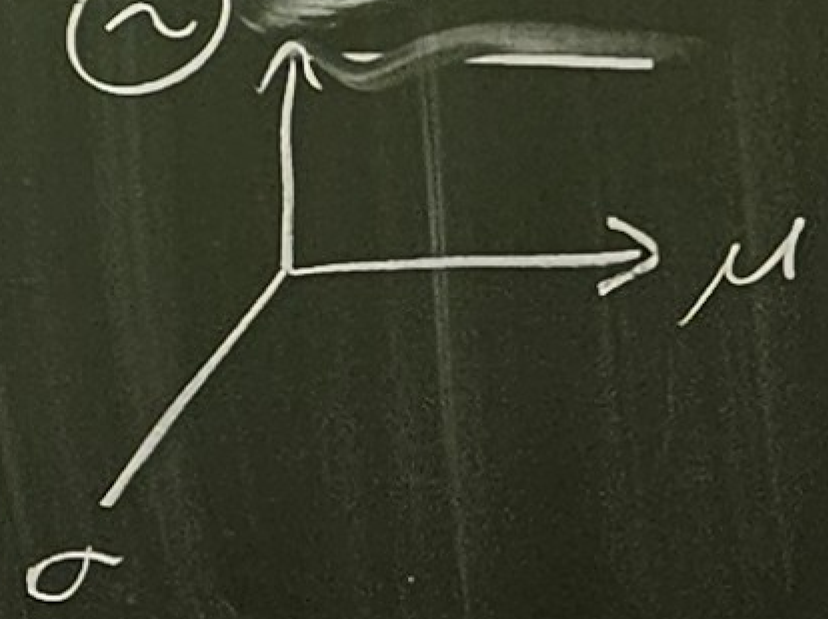
$$\ln p(\Theta) = \ln\left[\frac{1}{\sigma\sqrt{2\pi}}\right] + \left[-\frac{1}{2}\left(\frac{\Theta-\mu}{\sigma}\right)^2\right]$$

$$\frac{\partial \ln p(\Theta|x)}{\partial\Theta} = x\frac{1}{\Theta} - (N-x)\frac{1}{1-\Theta} - \frac{1}{2\sigma^2}\cdot 2(\Theta-\mu) = 0 \;/\cdot\Theta(1-\Theta)$$

$$x(\Theta-1) - (N-x) - \frac{\Theta-1}{2\sigma^2}$$

$$\Theta^3 - \Theta^2(\mu+1) - \Theta(N\sigma^2-\mu) + x\sigma^2 = 0$$

$\hat{\Theta}$



$\frac{1}{\sigma}$

$\Theta = \lambda$

$$P(x|\lambda) = \lambda e^{-\lambda x}$$

$$X = [x_1, x_2, \ldots x_N]$$

x_i - niezależne

$$L(\lambda) = \prod_{i=1}^{N} p(x_i|\lambda) = \prod_{i=1}^{N} \lambda e^{-\lambda x_i}$$

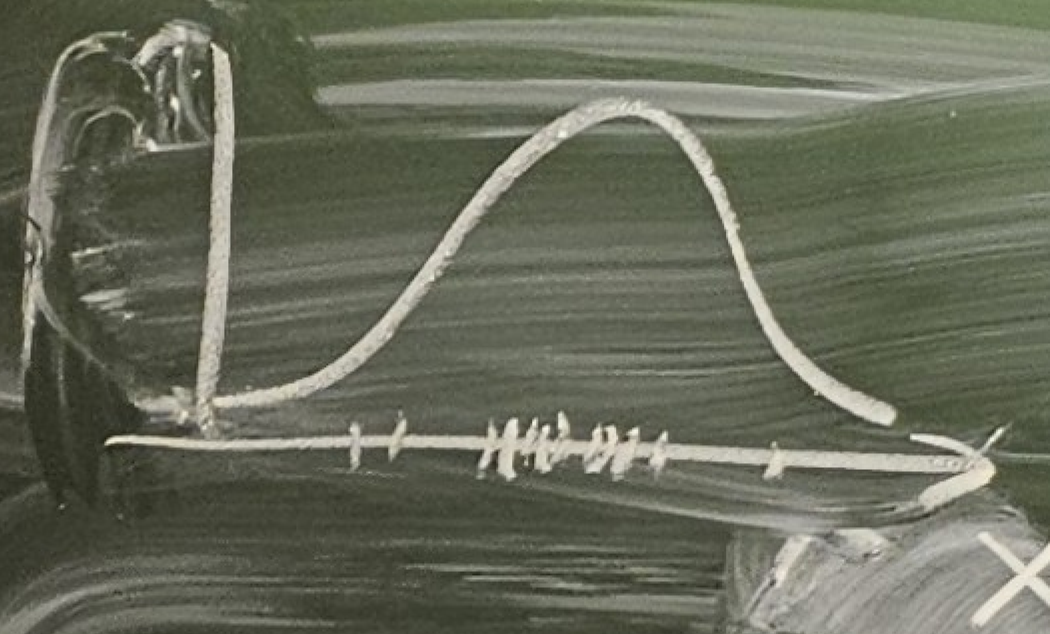
$$\ln L(\lambda) = \sum_{i=1}^{N} \ln(\lambda e^{-\lambda x_i}) = \sum_{i=1}^{N} \left(\ln\lambda + \ln(e^{-\lambda x_i})\right)$$

$$\ln L(\lambda) = N \ln\lambda - \lambda \sum_{i=1}^{N} x_i$$

$$\frac{\partial}{\partial\lambda} \ln L(\lambda) = \frac{N}{\lambda} - \sum_{i=1}^{N} x_i$$

$$\frac{\partial}{\partial\lambda} \ln L = 0 \Rightarrow \frac{N}{\lambda} = \sum_{i=1}^{N} x_i \Rightarrow \boxed{\hat{\lambda} = \frac{N}{\sum_{i=1}^{N} x_i}} = \frac{1}{\hat{x}}$$

$$\hat{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$



$$\Theta = \begin{bmatrix} \mu \\ \sigma \end{bmatrix}$$

$$P(x \mid \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

$$\underline{X} = [x_1 \; x_2 \; \dots \; x_N]$$

x_i - niezależne

$$\log L(\underline{X}, \mu, \sigma) = \sum_{i=1}^{N}\left(\log\left(\frac{1}{\sigma\sqrt{2\pi}}\right) - \frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right)$$

$$= -N\log\sigma - N\log\sqrt{2\pi} - \frac{1}{2\sigma^2}\sum_{i=1}^{N}(x_i-\mu)^2$$

$$\begin{cases} \dfrac{\partial L}{\partial \mu} = 0 \\ \dfrac{\partial L}{\partial \sigma} = 0 \end{cases} \Rightarrow \begin{array}{l} \hat{\mu} = ? \\ \hat{\sigma}^2 = ? \end{array}$$

w domu dokończyć

N rzutów monetą

Θ - praw. R — ln

X - ile razy R

$$P(X|\Theta) = \Theta^X (1-\Theta)^{N-X}$$

$$\parallel\!\!|$$

$$L(\Theta)$$

$X = [RORR]$ $\quad \Theta = \frac{1}{2}$

$$\log L(\Theta) = \ln(\theta)^X + \ln(1-\theta)^{N-X} = X \cdot \ln\theta + (N-X)\ln(1-\theta)$$

$$\frac{\partial P}{\partial \theta} = \frac{X}{\theta} - \frac{N-X}{1-\theta}$$

$$\frac{X}{\theta} - \frac{N-X}{1-\theta} = 0$$

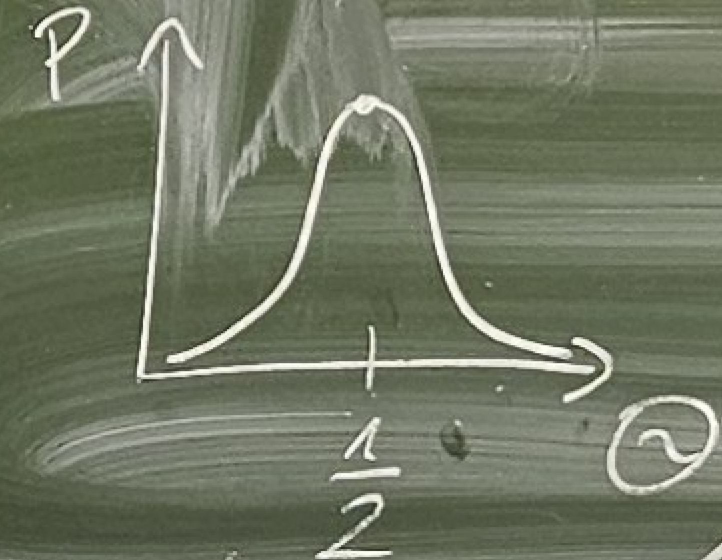
$$\frac{X}{\theta} = \frac{N-X}{1-\theta}$$

$$X(1-\theta) = \theta(N-X)$$

$$X - \cancel{\theta X} = \theta N - \cancel{\theta X}$$

$$X = \theta N$$

$$\boxed{\hat{\theta} = \frac{X}{N}}$$



$$\underbrace{P(\Theta|X)} = \overbrace{P(X|\Theta)}^{L} \cdot P(\Theta)$$

$$\log P(\Theta|X) = \log P(X|\Theta) + \log P(\Theta)$$

$$= x \log\Theta + (N-x)\log(1-\Theta)$$

$$P(\Theta) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\Theta-\mu}{\sigma}\right)^2\right]$$

$\mu, \sigma \doteq$ const.

$$-\log\sigma\sqrt{2\pi} - \frac{1}{2}\left(\frac{\Theta-\mu}{\sigma}\right)^2$$

N rzutów moneta

Θ - praw. R — M

X - ile razy R

$$P(X|\Theta) = \Theta^{X}(1-\Theta)^{N-X}$$

$$\parallel$$

$$L(\Theta)$$

$X = [RORR]$ $\Theta = \frac{1}{2}$

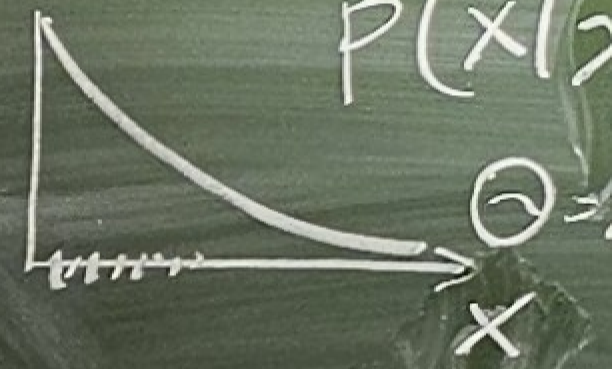
$$\frac{\partial P(\Theta|X)}{\partial \Theta} = \frac{X}{\Theta} - \frac{N-X}{1-\Theta} - \frac{\Theta-\mu}{\sigma^2} = 0$$

$$\vdots$$

$$\Theta^3 - \Theta^2(1-\mu) - \Theta(2X\sigma^2 + \mu + N\sigma^2) + X\sigma^2 = 0$$

$$P(\Theta) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\Theta-\mu}{\sigma}\right)^2\right]$$

μ, σ - const.



$$p(x|\lambda) = \lambda e^{-\lambda x}$$

$$\Theta = \lambda$$

$$X = [x_1 \; x_2 \; \dots \; x_N]$$

x_i - niezależne

$$p(X|\lambda) = \prod_{i=1}^{N} p(x_i|\lambda) = L(\lambda)$$

$$\log L(\lambda) = \sum_{i=1}^{N} \log \lambda e^{-\lambda x_i} = \sum_{i=1}^{N} \log \lambda - \sum_{i=1}^{N} \lambda x_i = N \log \lambda - \lambda \overbrace{\sum_{i=1}^{N} x_i}^{N \cdot \frac{1}{N}} = N \log \lambda - \lambda N \hat{x}$$

$$\hat{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$\frac{\partial \log L}{\partial \lambda} =$$

$$\boxed{\hat{\lambda} = \frac{1}{\hat{x}}}$$

W domu

$$p(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

$$\Theta = \begin{bmatrix} \mu \\ \sigma \end{bmatrix} \quad \begin{cases} \frac{\partial \log L}{\partial \mu} = 0 \\ \frac{\partial \log L}{\partial \sigma} = 0 \end{cases}$$