$$\frac{\partial}{\partial \Theta} \Theta x = x$$

$$\frac{\partial}{\partial \Theta_3} \sum_{j=1}^{5} \Theta_j x^{(j)} = x^{(3)}$$

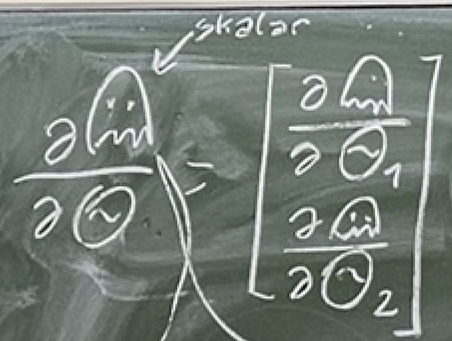
w domu

$$(A^T)^T \qquad (AB)^T = B^T A^T$$

Wykaz

$$(A^T)^{-1} = (A^{-1})^T \qquad (A \pm B)^T =$$

$$\Theta = \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix}$$



$$\frac{\partial}{\partial \Theta} \Theta^T x = \frac{\partial}{\partial \Theta}\left(\Theta_1 x^{(1)} + \Theta_2 x^{(2)}\right) = \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix} = x$$

$$(AB)^{-1} = B^{-1} A^{-1}$$

$$A^T \pm B^T$$

$$\frac{\partial}{\partial \Theta} \Theta^T x = x$$

$$\frac{\partial}{\partial \Theta} x = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\frac{\partial}{\partial \Theta} \Theta^T \Theta = 2\Theta$$

W domu

$$\frac{\partial}{\partial \Theta} \Theta^T A \Theta = ?$$

sym niesym

$\bar{y} = \Theta^T x$

$X_{K \times N}$

$\Theta^T X \qquad Y_{1 \times N}$

$\Theta_{K \times 1} \qquad x_{K \times 1} \qquad \bar{Y}_{1 \times N}$

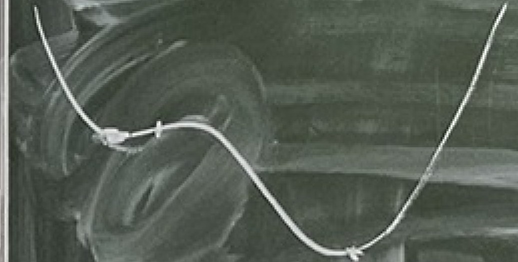
$E = Y - \bar{Y}$

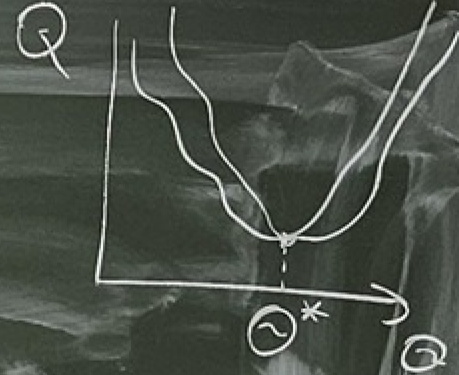
$\cancel{Q(\Theta) = \sum_{i=1}^{N} |e_i|}$ → numerycznie

$y_i - \bar{y}_i$

$\Theta^T x_i$

$$\frac{\partial}{\partial \Theta} Q = 0_K$$





$\epsilon = E^T$

$$Q(\Theta) = \sum_{i=1}^{N} e_i^2 = E E^T = \epsilon^T \epsilon$$

$$\frac{\partial Q}{\partial \Theta} = \frac{\partial}{\partial E}(E E^T) \cdot \frac{\partial}{\partial \Theta} E = 2E^T \frac{\partial}{\partial \Theta}(Y - \Theta^T X)$$

$$\frac{\partial}{\partial \Theta} \Theta^T x = x$$

$$\frac{\partial}{\partial \Theta} x = 0_K \qquad \frac{\partial}{\partial \Theta} \Theta^T \Theta = 2\Theta$$

$$\frac{\partial Q}{\partial \Theta} = 2\underbrace{E^T}_{N\times 1}\Big(\underbrace{\frac{\partial}{\partial \Theta} Y}_{0_{K\times N}} - \underbrace{\frac{\partial}{\partial \Theta}\Theta^T X}_{X}\Big) =$$

$$= 0_{K\times 1} - 2X E^T = -2X\left(Y^T - X^T\Theta\right) =$$

$$Q(\Theta) = \sum_{i=1}^{N} e_i^2 = E E^T =$$

$$\frac{\partial Q}{\partial \Theta} = \frac{\partial}{\partial E}\left(E E^T\right)\cdot\frac{\partial}{\partial \Theta} E = 2E^T \frac{\partial}{\partial \Theta}\left(Y - \Theta^T X\right) =$$

$$-2XY^T + 2XX^T\Theta = 0_K$$

$$XX^T\Theta = XY^T \;/\; (XX^T)^{-1}$$

$$\boxed{\Theta = (XX^T)^{-1} XY^T}$$

$$\frac{\partial}{\partial \Theta}\Theta^T x = x$$

$$\frac{\partial}{\partial \Theta} x = 0_K \qquad \frac{\partial}{\partial \Theta}\Theta^T\Theta = 2\Theta$$

$$\bar{y} = \Theta_1 x^{(1)} + \dots + \Theta_k x^{(k)}$$

$$\frac{d}{d\Theta_2} \Theta^T x = x^{(2)}$$

$$\frac{\partial}{\partial \Theta_2} \Theta^T x = x^{(2)}$$

$$\frac{\partial}{\partial \Theta} \Theta^T x = x$$

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$$\bar{y} = \Theta^T x \qquad \underset{K \times N}{X}, \underset{1 \times N}{Y} \qquad \bar{Y}$$

zmien. dec. Θ

Fcelu:

$$\bar{Y} = \Theta^T X$$

$$Y X^T = \Theta^T \underbrace{X X^T}_{K \times N \;\; N \times K}$$

$$\det(X) \neq 0$$

$$\Theta^T = Y X^T (X X^T)^{-1}$$

$$\Theta = (X X^T)^{-1} X Y^T$$

$$(AB)^{-1} = B^{-1} A^{-1}$$

$$(A^{-1})^T = (A^T)^{-1}$$

$$\frac{\partial}{\partial \Theta} \Theta^T x = x$$

$$\frac{\partial}{\partial \Theta} \Theta^T \Theta = [\theta_1 \dots \theta_k] \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_k \end{bmatrix} = \theta_1^2 + \theta_2^2 + \dots + \theta_k^2$$

$$= 2\Theta$$

$$(A^T)^T = A \qquad (AB)^T = B^T A^T$$

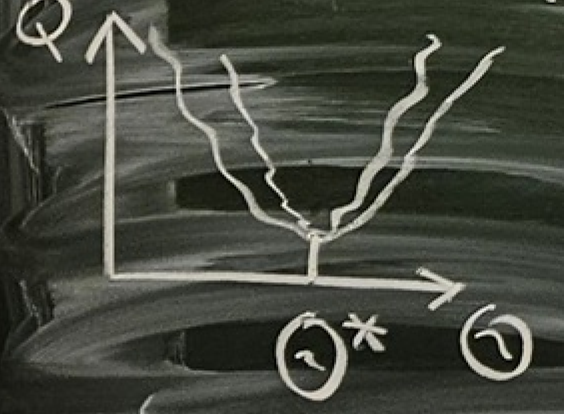
$$(A \pm B)^T = A^T \pm B^T$$

$$\bar{Y} = \Theta^T X$$

$$Q(\Theta) = \sum_{i=1}^{N} e_i^2 = E E^T$$

$$e_i = y_i - \bar{y}_i$$

$$\bar{y}_i = \Theta^T x_i$$



$$E = [e_1 \dots e_N] = Y - \bar{Y} = Y - \Theta^T X$$

$$\frac{\partial Q}{\partial \Theta} = 0_K$$

$$Q(E(\Theta))$$

$$\frac{\partial Q}{\partial \Theta} = \frac{\partial Q}{\partial E} \cdot \frac{\partial E}{\partial \Theta} = \left(\underbrace{\frac{\partial}{\partial E}\left(E E^T\right)}_{1 \times N} \cdot \underbrace{\left(\frac{\partial E}{\partial \Theta}\right)^T}_{K \times N} \right)^T$$

$$= \frac{\partial E}{\partial \Theta} \cdot \left(\frac{\partial}{\partial E} E E^T\right)^T = 0$$

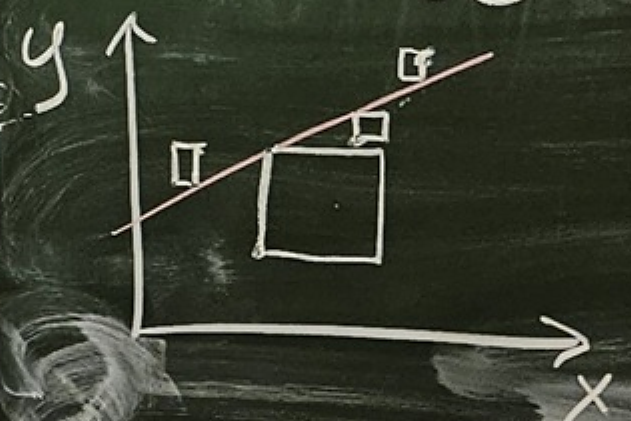
$$\Theta =$$

$$\frac{\partial}{\partial \Theta} \Theta^T x = x$$

$$\frac{\partial}{\partial \Theta} \Theta^T A \Theta = ?$$

W domu

$$\frac{\partial}{\partial \Theta} \Theta^T \Theta = 2\Theta$$



$$\bar{y}_i = \underset{(1\times K)}{\Theta^T} \underset{(K\times 1)}{x_i} \qquad \Theta_{K\times 1} \qquad x_{K\times 1}$$

$$\frac{\partial}{\partial \Theta_3}\Theta^T x = x^{(3)}$$

$$\frac{\partial}{\partial \Theta} x = 0_K$$

$$\frac{\partial}{\partial \Theta}\Theta^T x = x$$

$$\frac{\partial}{\partial \Theta}\Theta^T \Theta = 2\Theta$$

$$\frac{\partial}{\partial \Theta} x^T \Theta = x$$

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w domu: $\frac{\partial}{\partial \Theta}\Theta^T A \Theta$

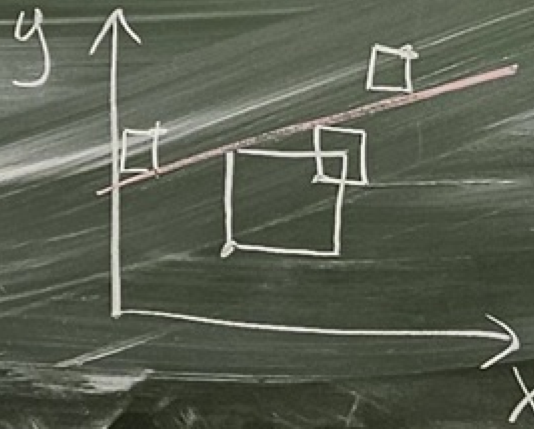
$$\underset{K\times N}{X} \qquad \underset{1\times N}{Y} \qquad \underset{1\times N}{\bar{Y}} \qquad \underset{1\times N}{E} = [e_1 \cdots e_N]$$

zm. dec: Θ

$$\bar{Y} = \Theta^T X$$

F celu: $Q(\Theta) = \frac{1}{N}\sum_{i=1}^{N} e_i^2$

$$\frac{\partial Q}{\partial \Theta} = 0_K$$



$$Q(\Theta) = \sum_{i=1}^{N} e_i^2 = EE^T$$

$$\frac{\partial}{\partial \Theta} EE^T = 0_K$$

$$\bar{y}_i = \underset{(1\times K)}{\Theta^T}\,\underset{(K\times 1)}{x_i} \qquad \Theta_{K\times 1} \qquad x_{K\times 1}$$

$$\frac{\partial}{\partial \Theta_3}\Theta^T x = x^{(3)}$$

$$\frac{\partial}{\partial \Theta} x = 0_K$$

$$\frac{\partial}{\partial \Theta}\Theta^T x = x$$

$$\frac{\partial}{\partial \Theta}\Theta^T \Theta = 2\Theta$$

$$\frac{\partial}{\partial \Theta} x^T \Theta = x$$

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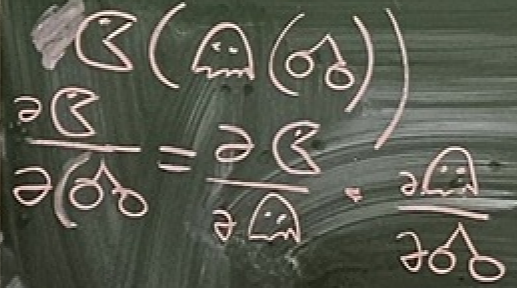
w domu

$$\frac{\partial}{\partial \Theta}\Theta^T A \Theta$$

$$\underset{K\times N}{X} \qquad \underset{1\times N}{Y} \qquad \underset{1\times N}{\bar{Y}} \qquad \underset{1\times N}{E} = [e_1 \cdots e_N]$$

$$\frac{3}{9} = \frac{3}{8}\cdot\frac{8}{7}\cdot\frac{7}{9}$$



$$\left(\left(\underbrace{\frac{\partial}{\partial E}EE^T}_{1\times N}\right)\cdot\left(\underbrace{\frac{\partial E}{\partial \Theta}}_{K\times N}\right)^T\right)^T = 0_K$$

$$\left(\frac{\partial E}{\partial \Theta}\right)\left(\frac{\partial}{\partial \Theta}EE^T\right)^T = 0_K$$

$$\left(\underbrace{\frac{\partial (Y-\Theta^T X)}{\partial \Theta}}_{K\times N}\right)\left(\underbrace{\cdot 2E^T}_{N\times 1}\right) = 0_K$$

$$Q(\Theta) = \sum_{i=1}^{N} e_i^2 = EE^T$$

$$(A^T)^{-1} = (A^{-1})^T$$

$$\frac{\partial}{\partial \Theta}\underbrace{EE^T}_{\text{w domu jako pochodna iloczynu}} = 0_K$$

$$(A\cdot B)^{-1} = B^{-1}A^{-1}$$

$$(A^T)^T = A$$

$$(A\cdot B)^T = B^T A^T$$

$$(A^{-1})^{-1} = A$$

$$(A\pm B)^T = A^T \pm B^T$$

$$\frac{\partial}{\partial \tilde{\Theta}_3} \tilde{\Theta}^T x = x^{(3)}$$

$$\frac{\partial}{\partial \tilde{\Theta}} x = 0_K$$

$$\frac{\partial}{\partial \tilde{\Theta}} \tilde{\Theta}^T x = x$$

$$\frac{\partial}{\partial \tilde{\Theta}} \tilde{\Theta}^T \tilde{\Theta} = = 2\tilde{\Theta}$$

$$\frac{\partial}{\partial \tilde{\Theta}} x^T \tilde{\Theta} = x$$

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$$\underset{K \times N}{X} \quad \underset{1 \times N}{Y} \quad \underset{1 \times N}{\bar{Y}} \quad \underset{1 \times N}{E} = [e_1 \cdots e_N]$$

$$(0_K - X) \cdot 2 \cdot (Y - \tilde{\Theta}^T X)^T = 0_K$$

$$\tilde{\Theta} =$$

$$\left(\frac{\partial E}{\partial \tilde{\Theta}}\right)\left(\frac{\partial}{\partial \tilde{\Theta}} E E^T\right)^T = 0_K$$

$$\underset{K \times N}{\left(\frac{\partial Y - \tilde{\Theta}^T X}{\partial \tilde{\Theta}}\right)} \underset{N \times 1}{\left(\cdot 2 E^T\right)} = 0_K$$