$$x_i = \begin{bmatrix} x_i^{(1)} \\ x_i^{(2)} \\ \vdots \\ x_i^{(K)} \end{bmatrix}$$

$$\{(x_i, y_i)\}_{i=1}^{N}$$

$$\frac{1}{N} E[\hat{X}\hat{X}^T]$$

$$y_i$$

$$\underbrace{\sigma_{jm}^2 = \frac{1}{N-1}\sum_{i=1}^{N} \hat{x}_i^{(j)} \hat{x}_i^{(m)}}$$

$$= \frac{1}{N-1} \underbrace{\hat{X}\hat{X}^T}_{K \times K}$$

$$X = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_N^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_N^{(2)} \\ \vdots & \vdots & & \\ x_1^{(K)} & x_2^{(K)} & \cdots & x_N^{(K)} \end{bmatrix}$$

$$Y = [y_1 \; y_2 \; \cdots \; y_N]$$

$$X = [x_1 \; x_2 \cdots x_N]$$

$$[x_1 - \mu \;\; x_2 - \mu \;\cdots\; x_N - \mu] = [x_1 \; x_2 \cdots x_N] - [\overbrace{\mu \;\; \mu \cdots \mu}^{N \text{ razy}}]$$

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i = \begin{bmatrix} \mu^{(1)} \\ \mu^{(2)} \\ \vdots \\ \mu^{(K)} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 \end{bmatrix}$$

$$X - \mu\underbrace{[1 \;\; 1 \cdots 1]}_{1_{1\times N}}$$

$$\hat{X} = \underbrace{X - \mu \cdot 1_{1\times N}}$$

$$\frac{1}{N-1}\sum_{i=1}^{N}(\overbrace{x_i^{(j)} - \mu^{(j)}}^{\hat{x}_i^{(j)}})^2 = \sigma_j^2$$

$$\frac{1}{N-1}\sum_{i=1}^{N}(x_i^{(j)} - \mu^{(j)})(x_i^{(m)} - \mu^{(m)}) = \sigma_{jm}^2$$

$$\bar{y} = \Theta^T x \qquad \Theta = \begin{bmatrix} \Theta_1 \\ \vdots \\ \Theta_K \end{bmatrix}$$

$$\bar{y} = \Theta_1 x^{(1)} + \Theta_2 x^{(2)} + \dots + \Theta_K x^{(K)} = \sum_{j=1}^{K} \Theta_j x^{(j)}$$

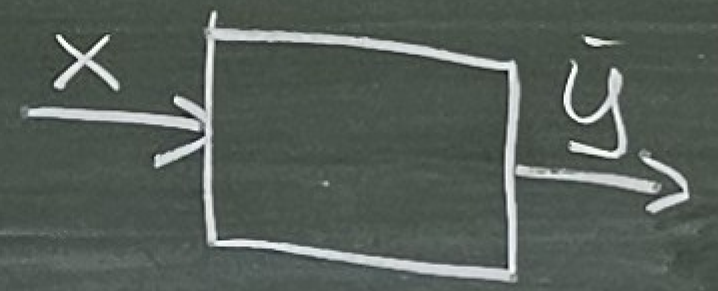
$$\bar{y} = ax^2 + bx + c$$

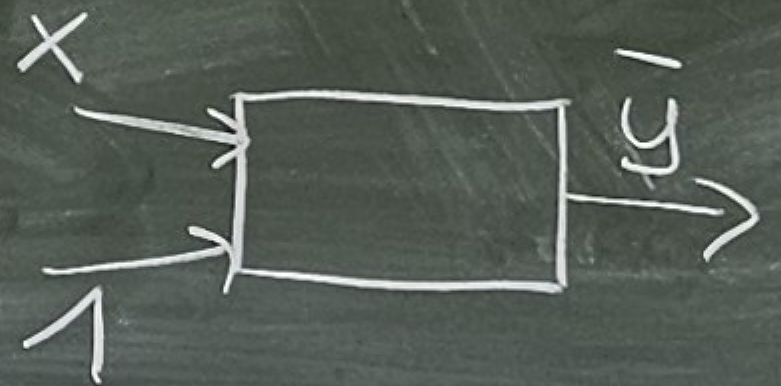
$$\Theta = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \qquad \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix} = \begin{bmatrix} x^2 \\ x \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \dots & x_N^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \dots & x_N^{(2)} \\ \vdots & \vdots & & \\ x_1^{(K)} & x_2^{(K)} & \dots & x_N^{(K)} \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 & y_2 & \dots & y_N \end{bmatrix}$$

$$x_i^{(j)}$$





$$X = \begin{bmatrix} 1 & 1 & 4 & 9 \\ 1 & -1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

i	1	2	3	4
x	1	-1	2	3
y	2	0	4	6

w domu

$$\bar{y} = ax + b$$

To nie jest model liniowy

$$\bar{y} = a x + b \, 1$$

$$\bar{y} = \Theta_1 x^{(1)} + \Theta_2 x^{(2)}$$

$$\Theta = \begin{bmatrix} a \\ b \end{bmatrix} \qquad x \leftarrow \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix} = \begin{bmatrix} x \\ 1 \end{bmatrix}$$

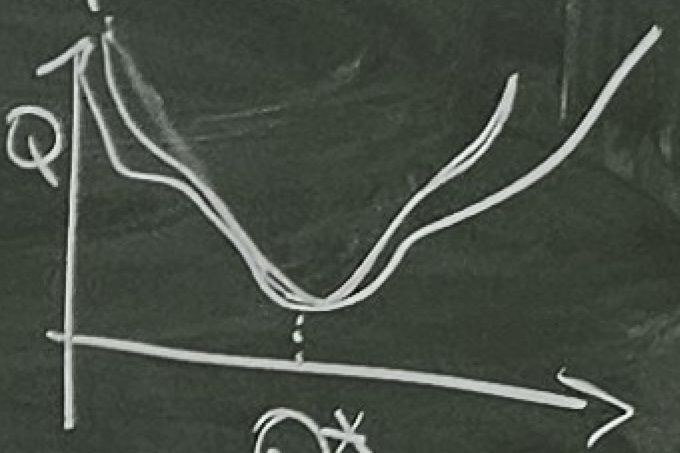
$$\bar{y}_i = \Theta^T x_i$$

zm. dec

$$Q = \frac{1}{N}\sum_{i=1}^{N} |e_i|$$

F celu

$$Q(\Theta) =$$



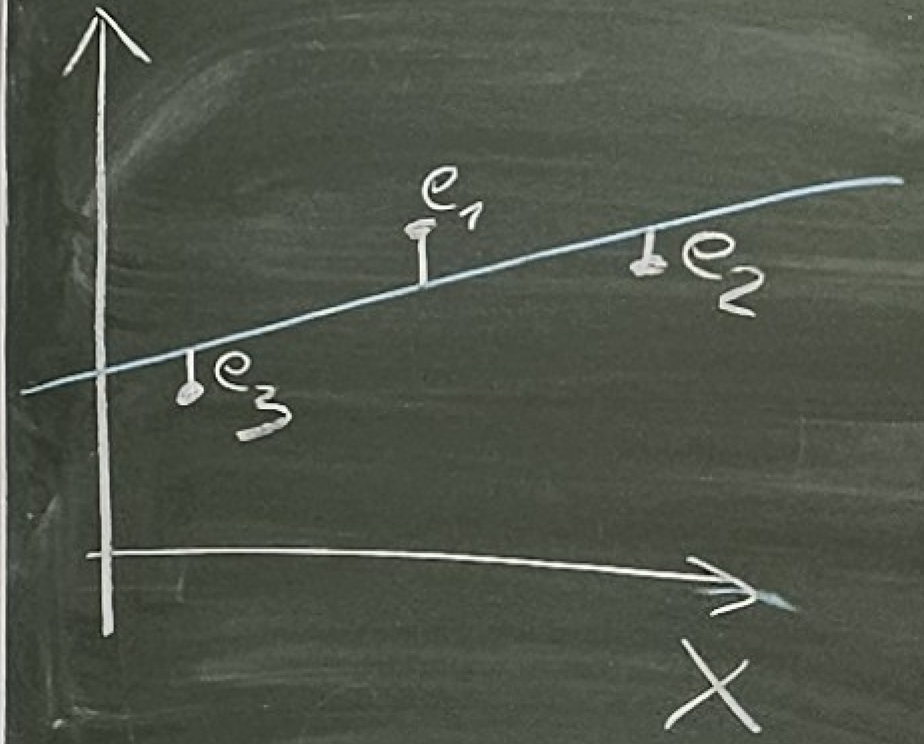
$$\Theta^* = \arg\min_{\Theta} Q(\Theta)$$

$$X = \begin{bmatrix} X_1^{(1)} & X_2^{(1)} & \dots & X_N^{(1)} \\ X_1^{(2)} & X_2^{(2)} & \dots & X_N^{(2)} \\ \vdots & \vdots & & \vdots \\ X_1^{(K)} & X_2^{(K)} & \dots & X_N^{(K)} \end{bmatrix}$$

$$\left.\begin{aligned} Y &= [y_1 \quad y_2 \quad \dots \quad y_N] \\ \bar{Y} &= [\bar{y}_1 \quad \bar{y}_2 \quad \dots \quad \bar{y}_N] \end{aligned}\right\} \text{aproksymacja}$$

$$e_i = y_i - \bar{y}_i$$

$$E = [e_1 \quad e_2 \quad \dots \quad e_N] = Y - \bar{Y}$$



$$X = \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ \vdots \\ x^{(k)} \end{bmatrix} \quad \{(x_i, y_i)\}$$

$$\Updownarrow$$

$$y \qquad \mathbb{X}, \mathbb{Y}$$

$$N\left\{ \mathbb{X} = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \dots & x_N^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \dots & x_N^{(2)} \\ \vdots & \vdots & & \\ x_1^{(k)} & x_2^{(k)} & \dots & x_N^{(k)} \end{bmatrix} \right.$$

$i=1$

$$\mathbb{Y} = [\, y_1 \quad y_2 \quad \cdots \quad y_N \,]$$

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \mu = \begin{bmatrix} \mu^{(1)} \\ \mu^{(2)} \\ \vdots \\ \mu^{(k)} \end{bmatrix}$$

$$\mathbb{X} - \mu \qquad x_i - \mu$$

$$[\, x_1 - \mu \quad x_2 - \mu \quad \cdots \quad x_N - \mu \,] = \mathbb{X} - [\, \underbrace{\mu \cdots \mu}_{N \text{ razy}} \,] = \mathbb{X} - \mu\,[\, \underbrace{1 \cdots 1}_{N \text{ razy}} \,] = \mathbb{X} - \mu \cdot \mathbb{1}_N$$

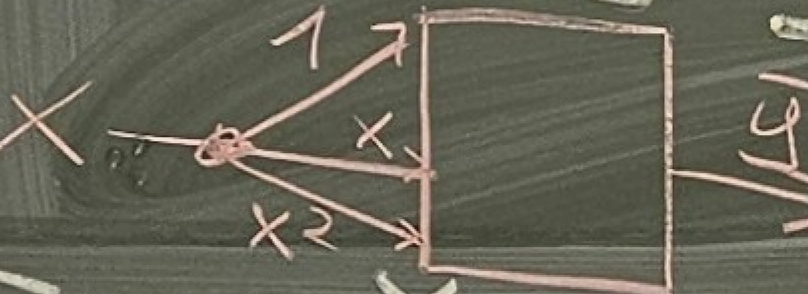
$$X = \begin{bmatrix} X^{(1)} \\ X^{(2)} \\ \vdots \\ X^{(K)} \end{bmatrix} \qquad \Theta = \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \vdots \\ \Theta_K \end{bmatrix}$$

$$\bar{y} = \Theta^T X = \Theta_1 X^{(1)} + \Theta_2 X^{(2)} + \dots + \Theta_K X^{(K)} = \sum_{j=1}^{K} \Theta_j X^{(j)}$$

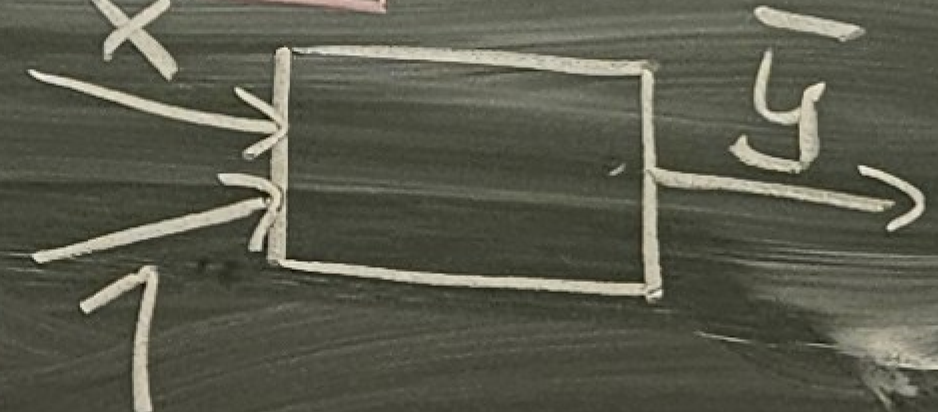
$$\bar{y} = a X^{(1)} + b X^{(2)} \qquad X = \begin{bmatrix} X^{(1)} \\ X^{(2)} \end{bmatrix} \qquad \Theta = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\mathbb{X} = \begin{bmatrix} X_1^{(1)} & X_2^{(1)} & \dots & X_N^{(1)} \\ X_1^{(2)} & X_2^{(2)} & \dots & X_N^{(2)} \\ \vdots & \vdots & & \\ X_1^{(K)} & X_2^{(K)} & \dots & X_N^{(K)} \end{bmatrix}$$

$$\mathbb{Y} = [\, y_1 \ \ y_2 \ \dots \ y_N \,]$$



Odp.



$$\begin{bmatrix} X^{(1)} \\ X^{(2)} \end{bmatrix} = \begin{bmatrix} X \\ 1 \end{bmatrix} \qquad \Theta = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\bar{y} = c + bx + ax^2$$

$$\begin{bmatrix} 1 \\ x \\ x^2 \end{bmatrix} = \begin{bmatrix} X^{(1)} \\ X^{(2)} \\ X^{(3)} \end{bmatrix} \qquad \Theta = \begin{bmatrix} c \\ b \\ a \end{bmatrix}$$

x	2	-1	3	0
y	3	1	8	-1

$$\mathbb{X} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 3 & 0 \\ 4 & 1 & 9 & 0 \end{bmatrix}$$

$$\bar{y} = ax + b$$

W domu

To nie jest model liniowy!

$$\bar{y} = ax + b \cdot 1$$

$$\bar{y} = \Theta_1 x^{(1)} + \Theta_2 x^{(2)}$$

$$X = \begin{bmatrix} X^{(1)} \\ X^{(2)} \\ \vdots \\ X^{(K)} \end{bmatrix} \qquad \Theta = \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \vdots \\ \Theta_K \end{bmatrix}$$

$$\bar{y} = \Theta^T X$$

zm. dec.: Θ

F. celu: $Q(\Theta) = \frac{1}{N}\sum_{i=1}^{N} e_i$

$$\underline{X} = \begin{bmatrix} X_1^{(1)} & X_2^{(1)} & \cdots & X_N^{(1)} \\ X_1^{(2)} & X_2^{(2)} & \cdots & X_N^{(2)} \\ \vdots & \vdots & & \vdots \\ X_1^{(K)} & X_2^{(K)} & \cdots & X_N^{(K)} \end{bmatrix}$$

$$\underline{Y} = [\, y_1 \;\; y_2 \;\; \cdots \;\; y_N \,]$$

$$\underline{\bar{Y}} = [\, \bar{y}_1 \;\; \bar{y}_2 \;\; \cdots \;\; \bar{y}_N \,]$$

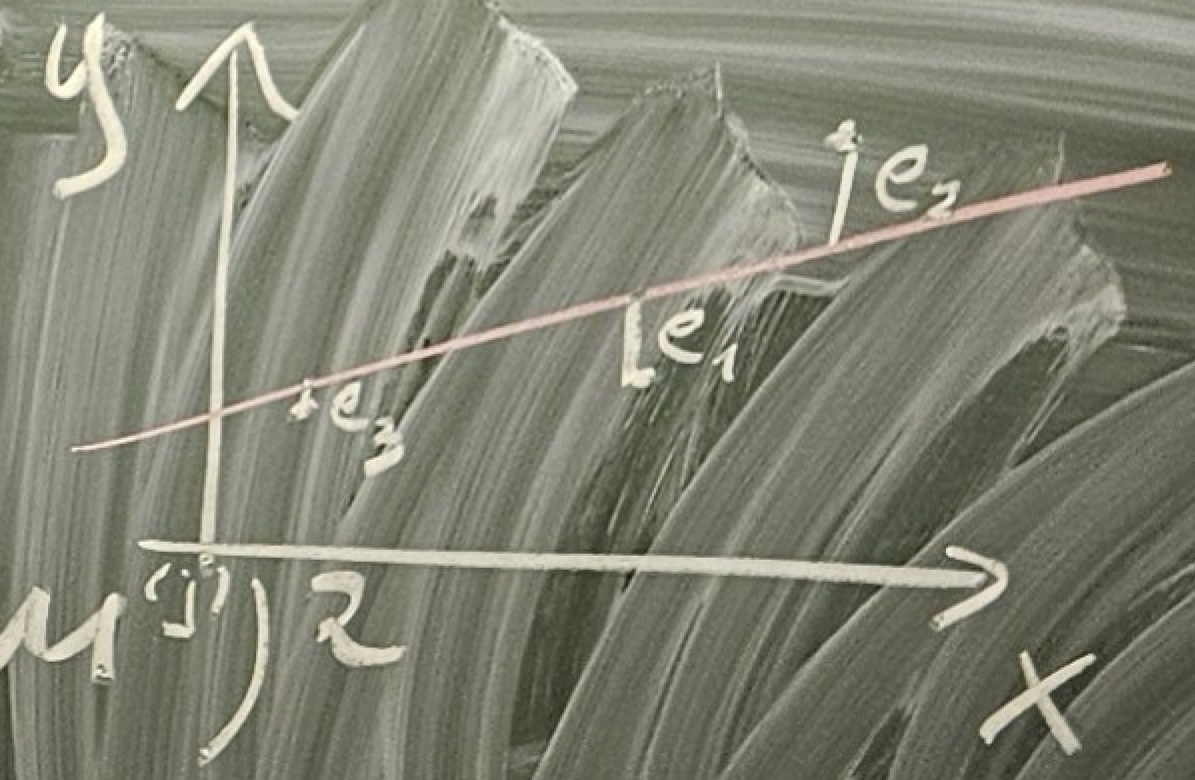
$$\underline{E} = [\, e_1 \;\; e_2 \;\; \cdots \;\; e_N \,]$$

$$e_i = y_i - \bar{y}_i$$

$$Var(x^{(j)}) = \frac{1}{N-1}\sum_{i=1}^{N} (x_i^{(j)} - \mu^{(j)})^2$$

$$\sigma_{jm}^2 = \frac{1}{N-1}\sum_{i=1}^{N} \underbrace{(x_i^{(j)} - \mu^{(j)})}_{\hat{x}_i^{(j)}} \underbrace{(x_i^{(m)} - \mu^{(m)})}_{\hat{x}_i^{(m)}}$$

$$\hat{X} \qquad \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \cdots & \sigma_{1K}^2 \\ \vdots & & & \vdots \\ \sigma_{K1}^2 & \cdots & & \sigma_{KK}^2 \end{bmatrix}$$



$$x = \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ \vdots \\ x^{(K)} \end{bmatrix} \qquad \{(x_i, y_i)\}_{i=1}^{N}$$

$$X, Y$$

$$y$$

$$X = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_N^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_N^{(2)} \\ \vdots & \vdots & & \\ x_1^{(K)} & x_2^{(K)} & \cdots & x_N^{(K)} \end{bmatrix}$$

$$Y = [\, y_1 \quad y_2 \quad \cdots \quad y_N \,]$$

$$\hat{X}\hat{X}^T$$

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \mu = \begin{bmatrix} \mu^{(1)} \\ \vdots \\ \mu^{(K)} \end{bmatrix}$$

$$\underset{K\times N}{\hat{X}} = [\, x_1 - \mu \ \cdots \ x_N - \mu \,]$$

$$= [\, x_1 \cdots x_N \,] - [\, \underbrace{\mu \cdots \mu}_{N} \,]$$

$$= \underset{K\times N}{X} - \mu[\, \underbrace{1 \cdots 1}_{N} \,] =$$

$$= X - \mu \cdot 1_N^T$$

$$\frac{1}{N-1}\sum_{i=1}^{N} (x_i^{(j)} - \mu^{(j)})^2 = Var(x^{(j)})$$

$$\frac{1}{N-1}\sum_{i=1}^{N} \underbrace{(x_i^{(j)} - \mu^{(j)})}\underbrace{(x_i^{(m)} - \mu^{(m)})}$$

$$= \sigma_{jm}^2 \qquad \hat{x}_i^{(j)} \cdot \hat{x}_i^{(m)}$$

$$\begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \cdots & \sigma_{1K}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2K}^2 \\ \vdots & & & \\ \sigma_{K1}^2 & \sigma_{K2}^2 & \cdots & \sigma_{KK}^2 \end{bmatrix}_{K\times K}$$

$$x = \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ \vdots \\ x^{(K)} \end{bmatrix} \qquad \Theta = \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \vdots \\ \Theta_K \end{bmatrix}$$

$$\bar{y} = \Theta^T x = \sum_{j=1}^{K} \Theta_j x^{(j)} =$$

$$= \Theta_1 x^{(1)} + \Theta_2 x^{(2)} + \ldots + \Theta_K x^{(K)}$$

$$\Theta = \begin{bmatrix} a \\ b \end{bmatrix} \qquad x^{(1)} = x \qquad x^{(2)} = 1$$

$$X = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_N^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_N^{(2)} \\ \vdots & \vdots & & \\ x_1^{(K)} & x_2^{(K)} & \cdots & x_N^{(K)} \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 & y_2 & \cdots & y_N \end{bmatrix}$$

$$\bar{y} = a x^3 + b x + c$$

$$\bar{y} = \Theta_1 x^{(1)} + \Theta_2 x^{(2)} + \Theta_3 x^{(3)}$$

$$\Theta = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \qquad x^{(1)} = x^3 \qquad x^{(2)} = x \qquad x^{(3)} = 1$$

x	1	-1	2	3
y	2	0	4	13

$$X = \begin{bmatrix} \quad \end{bmatrix}$$

$$\bar{y} = ax + b$$

W domu:

To nie jest model liniowy!

$$\bar{y} = a x + b \; 1$$

$$\bar{y} = \Theta_1 x^{(1)} + \Theta_2 x^{(2)}$$

$$x = \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ \vdots \\ x^{(K)} \end{bmatrix} \quad \Theta = \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \vdots \\ \Theta_K \end{bmatrix}$$

$$\bar{y} = \Theta^T x$$

zm. dec.: Θ

$$e_i = y_i - \bar{y}_i$$

$$X = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_N^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_N^{(2)} \\ \vdots & \vdots & & \vdots \\ x_1^{(K)} & x_2^{(K)} & \cdots & x_N^{(K)} \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 & y_2 & \cdots & y_N \end{bmatrix}$$

$$\bar{Y} = \begin{bmatrix} \bar{y}_1 & \bar{y}_2 & \cdots & \bar{y}_N \end{bmatrix}$$

$$E = \begin{bmatrix} e_1 & e_2 & \cdots & e_N \end{bmatrix}$$

F. celu:

$$Q(\Theta) = \sum_{i=1}^{N} |e_i|$$

$$4^2 + (-8)^2 + (-23)^2$$

$$|4| + |-8| + |-23|$$

$$5x^2 - 2x - 3 = 0$$