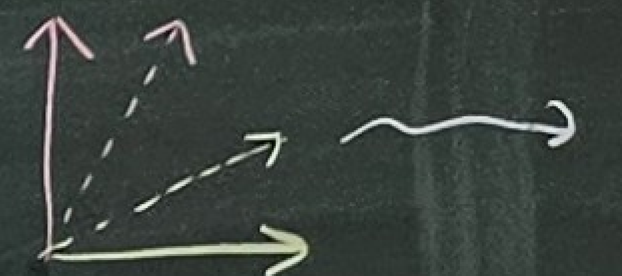


K

$P = [\quad]$

$$P v = \lambda v$$

$$P v - \lambda v = 0_K \qquad \lambda \in \mathbb{R}$$

$$I P v - I \lambda v = 0_K$$

$$(I P - I \lambda) v = 0_K$$

$$(P - I \lambda) v = 0_K$$

$$v \neq 0_K$$

$$P = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$$

$$\boxed{\det(P - I\lambda) = 0}$$

$$\lambda^2 - 2\lambda + \frac{3}{4} = 0$$

$$\lambda_1 = \frac{1}{2} \qquad \lambda_2 = \frac{3}{2}$$

$v_1 =$

$v_2 =$

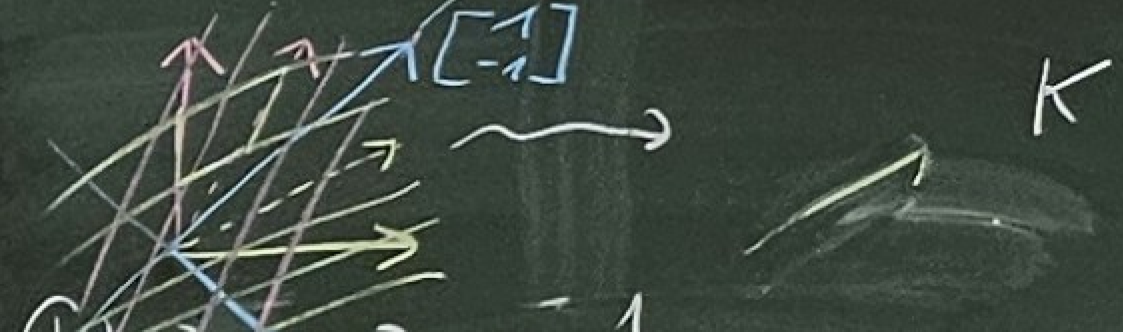
numpy → linalg.eig(P)

$$\begin{array}{cc} \lambda_1 & \lambda_2 \\ \end{array}$$

$$[\, 0.5 \quad 1.5 \,]$$

$$\begin{array}{cc} v_1 & v_2 \\ \end{array}$$

$$\begin{bmatrix} 0.7 & -0.5 \\ 0.3 & 0.5 \end{bmatrix}$$



$\lambda = \lambda_1 = \frac{1}{2}$

$\frac{1}{2} v^{(1)} + \frac{1}{2} v^{(2)} = 0$

$v^{(1)} = -v^{(2)}$

$v_1 = \begin{bmatrix} v^{(1)} \\ -v^{(1)} \end{bmatrix} = \underbrace{v^{(1)}}_{\in \mathbb{R}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$P = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$

$\begin{bmatrix} 1-\lambda & \frac{1}{2} \\ \frac{1}{2} & 1-\lambda \end{bmatrix} \begin{bmatrix} v^{(1)} \\ v^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{cases} (1-\lambda) v^{(1)} + \frac{1}{2} v^{(2)} = 0 \\ \frac{1}{2} v^{(1)} + (1-\lambda) v^{(2)} = 0 \end{cases}$

λ

$\underbrace{(P - I\lambda)} v = 0_K$

$\lambda_1 = \frac{1}{2}$ $\qquad$ $\lambda_2 = \frac{3}{2}$

$v_1 = \begin{bmatrix} v_1^{(1)} \\ v_1^{(2)} \end{bmatrix}$ $\qquad$ $v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} v^{(1)}$

$(x-1)(x-2)$

$P = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$P \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$\lambda_1 =$

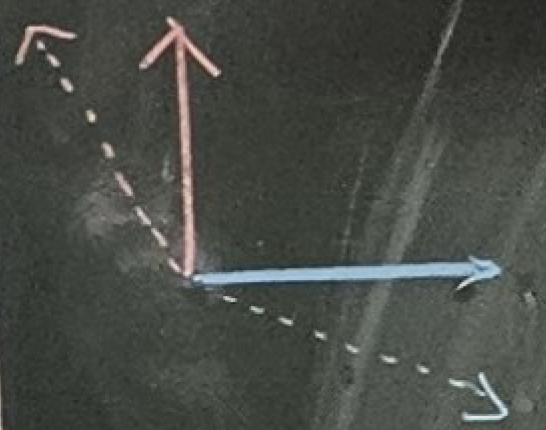
$\lambda_2 =$

$$P v_1 = \lambda_1 v_1$$

$$P v_2 = \lambda_1 v_2$$

$$\left\{ \begin{array}{l} V \leftarrow v_1,\ v_2 \\ D \leftarrow \lambda_1,\ \lambda_2 \end{array} \right\}$$

jedno równ.
macierzowe
z P, V, D



$$P = \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$$\underline{Pv = \lambda v}$$

$\lambda \in \mathbb{R}$

$$v \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$Pv - \lambda v = 0_k \quad / \cdot I_{k \times k}$$

$$IPv - I\lambda v = 0_k$$

$$(P - \lambda I) v = 0_k$$

$$\det(P - \lambda I) = 0$$

$$\begin{bmatrix} 5 & 8 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

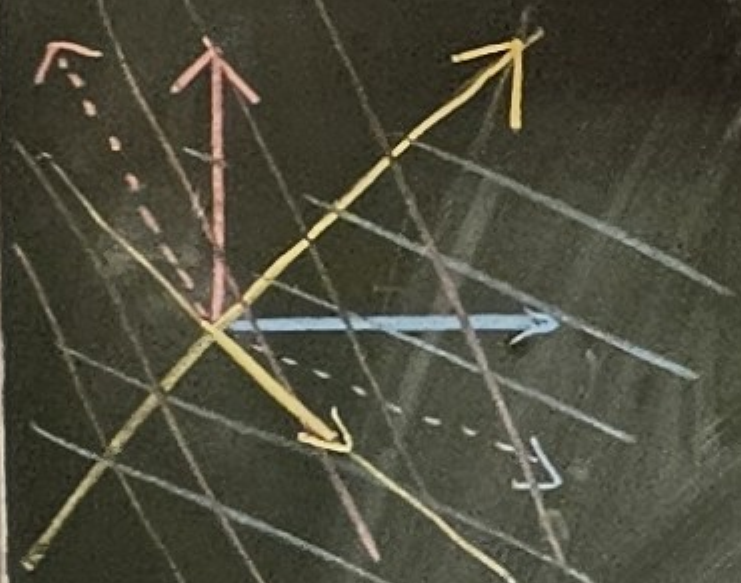
$\det(\ )$

$$v = \begin{bmatrix} v^{(1)} \\ v^{(2)} \\ \vdots \\ v^{(k)} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} =$$

$$= \begin{bmatrix} 1-\lambda & -\frac{1}{2} \\ -\frac{1}{2} & 1-\lambda \end{bmatrix}$$



$$P=\begin{bmatrix}1 & -\frac{1}{2}\\ -\frac{1}{2} & 1\end{bmatrix}$$

$$\underline{Pv=\lambda v}$$

$$\lambda\in\mathbb{R}$$

$$v\neq\begin{bmatrix}0\\0\end{bmatrix}$$

$$\det\begin{pmatrix}1-\lambda & -\frac{1}{2}\\ -\frac{1}{2} & 1-\lambda\end{pmatrix}=$$

$$=(1-2\lambda+\lambda^2)-\left(\frac{1}{4}\right)=$$

$$=\frac{3}{4}-2\lambda+\lambda^2=0$$

$$\det(P-\lambda I)=0$$

$$\Delta=4-4\cdot\frac{3}{4}=4-3=1$$

$$\sqrt{\Delta}=1$$

$$(P-\lambda I)\begin{bmatrix}v^{(1)}\\v^{(2)}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}1-\lambda & -\frac{1}{2}\\ -\frac{1}{2} & 1-\lambda\end{bmatrix}\begin{bmatrix}v^{(1)}\\v^{(2)}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\lambda_1=\frac{2-1}{2}=\frac{1}{2}\quad \mathrm{I}^\circ$$

$$\lambda_2=\frac{2+1}{2}=\frac{3}{2}\quad \mathrm{II}^\circ$$

$$\begin{bmatrix}(1-\lambda)v^{(1)}-\frac{1}{2}v^{(2)}\\ -\frac{1}{2}v^{(1)}+(1-\lambda)v^{(2)}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

λ_1 numpy → linalg.eig

$$\begin{bmatrix}\frac{1}{2}v^{(1)}-\frac{1}{2}v^{(2)}\\ \cancel{-\frac{1}{2}v^{(1)}+\frac{1}{2}v^{(2)}}\end{bmatrix}=\begin{bmatrix}0\\ \cancel{0}\end{bmatrix}$$

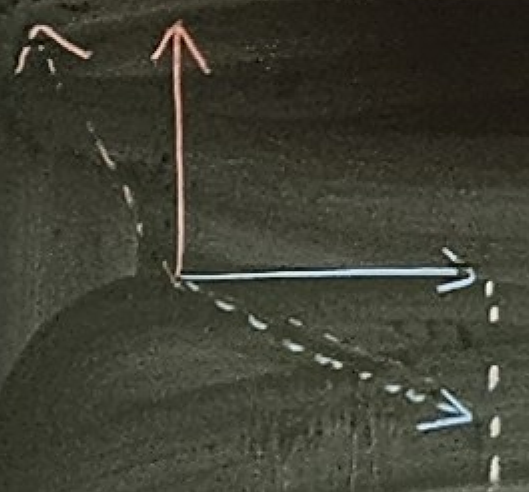
$$\frac{1}{2}v^{(1)}-\frac{1}{2}v^{(2)}=0$$

$$v^{(1)}=v^{(2)}$$

$$\begin{bmatrix}v^{(1)}\\v^{(1)}\end{bmatrix}$$

$\frac{1}{2}$

$\frac{1}{2}$



$$P = \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underbrace{Pv = \lambda v}$$

$$v \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \lambda \in \mathbb{R}$$

$$v = \begin{bmatrix} v^{(1)} \\ v^{(2)} \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

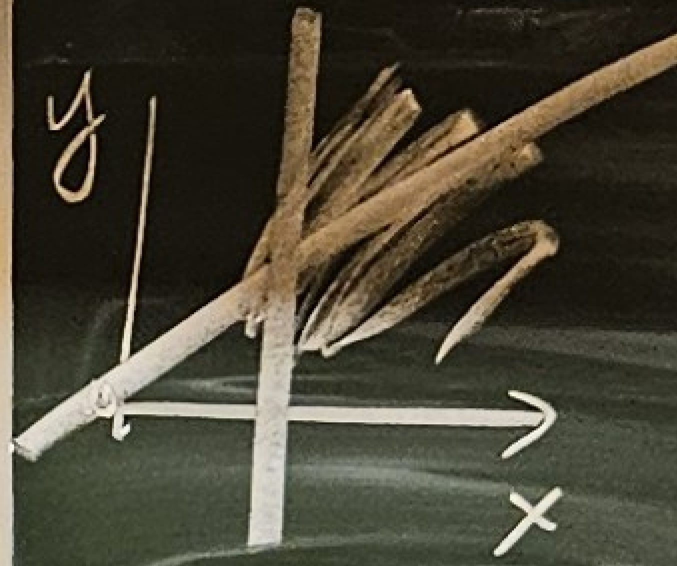
$$\begin{bmatrix} -1 & 5 & 8 \\ 3 & 1 & 2 \\ 4 & -4 & -6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} & 2 & \\ & 1 & \\ & 3 & \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 3 & 4 \\ 5 & 1 & -4 \\ 8 & 2 & -6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



$2x + 3y = 0$

jak przetestować, czy któryś wiersz/ kolumnę dały się wyrazić jako suma ważona pozostałych ?