

$$w = \begin{bmatrix} x \\ y \\ F \end{bmatrix} \quad A = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix}$$

$$\begin{bmatrix} x & y & F \end{bmatrix} \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix} \begin{bmatrix} x \\ y \\ F \end{bmatrix} =$$

$$x^2 A + y^2 B + c F^2$$

$$A = a$$

$$B = c$$

$$C = xy$$

$$F = \sqrt{b}$$

$$ax^2 + bxy + cy^2 = w^T A w$$

$$A = \text{sym} \\ A^T = A$$

$$w = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$A \in \mathbb{R}^{2 \times 2}$$

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

$$A = \begin{bmatrix} a & b/2 \\ b/2 & c \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$



$$w = \begin{bmatrix} x & y \end{bmatrix} \quad A = \begin{bmatrix} a & 0 & 0 \\ 0 & bxy & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$w = \begin{bmatrix} x \\ \sqrt{xy} \\ y \end{bmatrix}$$

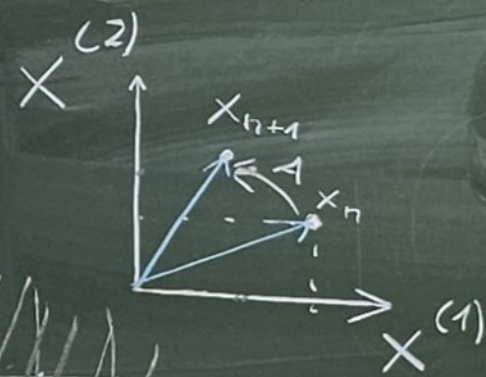
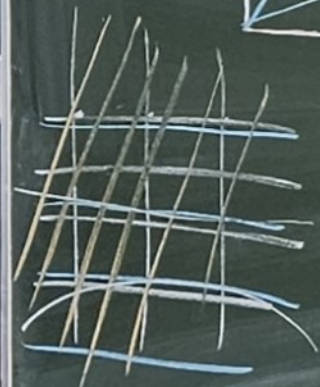
$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$\forall x, y \in \mathbb{C}$$

$$x_{n+1} = A x_n$$

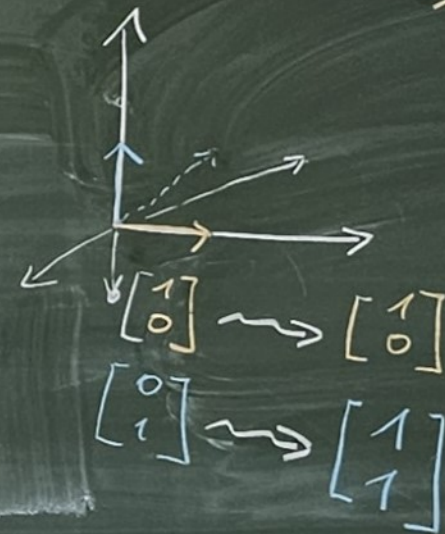
$$\dot{x} = a x(t)$$

$$x_3 = \begin{bmatrix} x_3^{(1)} \\ x_3^{(2)} \end{bmatrix}$$



$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

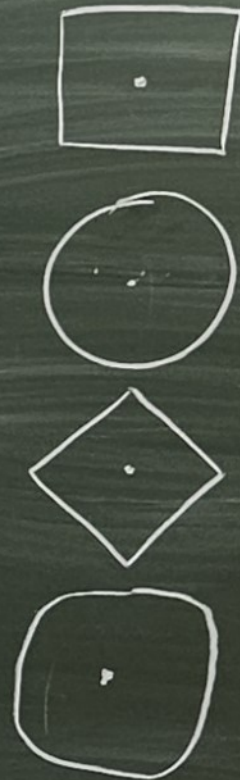
$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix}$$



$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

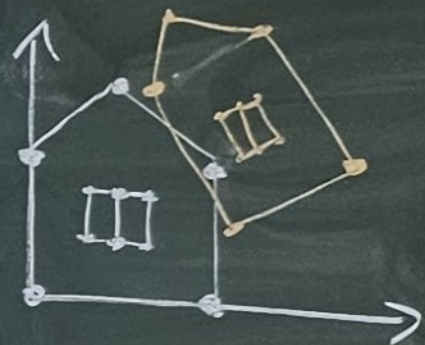
$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

A
w domu
macierz
obrotu
o kąt
 α



W domu

$$A \left[\begin{pmatrix} x_1^{(1)} \\ x_1^{(2)} \end{pmatrix} \begin{pmatrix} x_2^{(1)} \\ x_2^{(2)} \end{pmatrix} \dots \right]$$



$\det(A)$

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



ściananie
obrót $\begin{pmatrix} A_1 \cdot x \\ A_2 \cdot x \end{pmatrix}$

obrót
ściananie

$$A_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$C = A_2 \cdot A_1$$

$$D = A_1 \cdot A_2$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A_1 \cdot ? = I$$

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 \end{array} \right]$$

$$A_1^{-1}$$

$$ax^2 + bxy + cy^2 = \underbrace{w}_{1 \times 2}^T \underbrace{A}_{2 \times 2} \underbrace{w}_{2 \times 1}$$

$$w = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & \Delta \end{bmatrix}$$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & \Delta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\alpha + y\gamma & x\beta + y\Delta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= x(x\alpha + y\gamma) + y(x\beta + y\Delta) = x^2\alpha + xy(\gamma + \beta) + y^2\Delta$$

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

$$\begin{bmatrix} a & b/2 \\ b/2 & c \end{bmatrix} \quad A^T = A$$

$$\begin{bmatrix} x\beta + y\Delta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$$

$$ax^2 + bxy + cy^2 = \underbrace{w}_{1 \times 2}^T \underbrace{A}_{2 \times 2} \underbrace{w}_{2 \times 1}$$

$$w = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$A = \begin{bmatrix} a & b/2 \\ b/2 & c \end{bmatrix}$$

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & bxy & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} x \\ 1 \\ y \end{bmatrix} = w$$

$$\begin{bmatrix} x & 1 & y \end{bmatrix} \begin{bmatrix} ax & bxy & cy \end{bmatrix} \begin{bmatrix} ax^2 + bxy + cy^2 \end{bmatrix}$$

$$A^T = A$$

$$w = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$w = \begin{bmatrix} x \\ 0 \\ y \end{bmatrix}$$

$$x_{n+1} = A x_n$$

oni tego nie
kojarzą

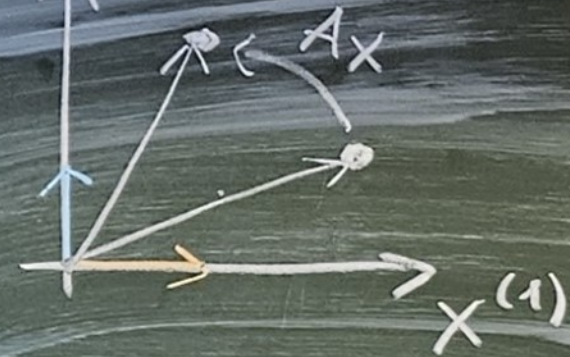
$$B \cdot ? = I$$



$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$x^{(2)}$



$$x = \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix}$$

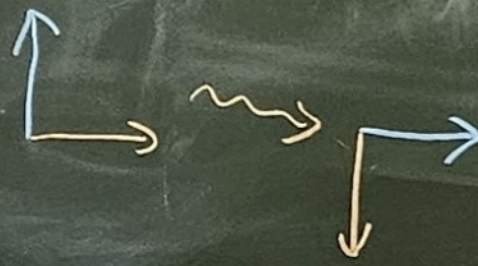
Ax

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$



w domu
macierz obrotu
o kąt α

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ -1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

x
skalowania A

obrot B

$$\begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$Ax \stackrel{?}{=} xA$$

x

obrot B

B -wynik

$$B \cdot A \cdot x$$

$$A \cdot B \cdot x$$

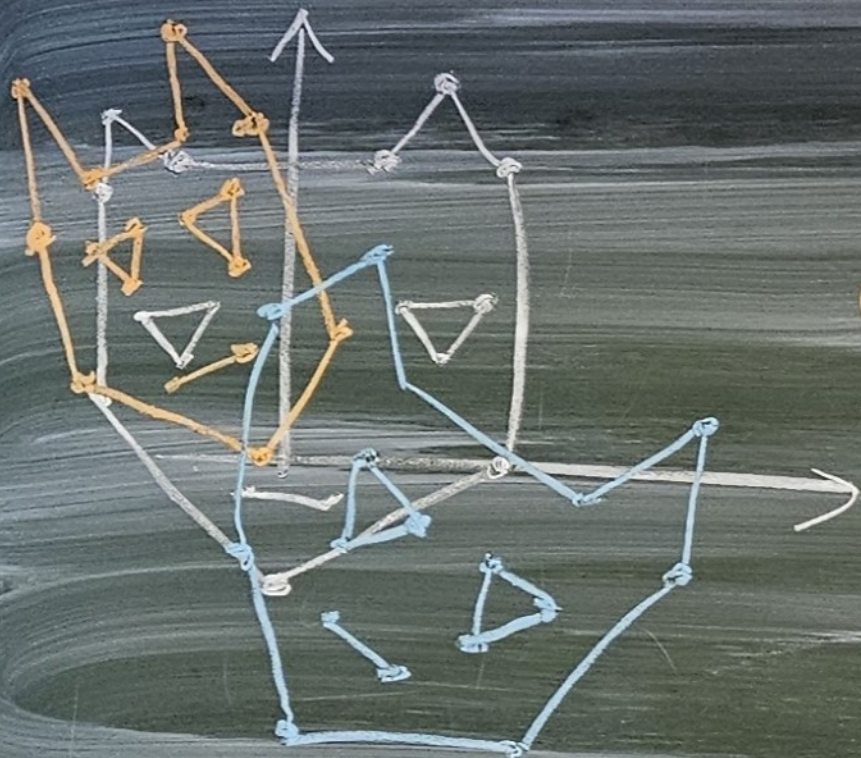
skalowania A

$$B \cdot A = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} -1 & 2 \\ -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

w domu



A
 $\det(A)$

B

$\det(B)$

A

$\begin{bmatrix} X_1^{(1)} \\ X_1^{(2)} \end{bmatrix}$

$\begin{bmatrix} X_2^{(1)} \\ X_2^{(2)} \end{bmatrix}$

$\begin{bmatrix} X \\ // \\ \end{bmatrix}$

$$ax^2 + bxy + y^2c = w^T A w$$

$$w = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$A = \begin{bmatrix} \alpha & \beta \\ \beta & d \end{bmatrix}$$

$$\begin{bmatrix} \alpha & b \\ 0 & c \end{bmatrix}$$

$$\beta + \beta = b$$

$$\begin{bmatrix} \alpha & b/2 \\ b/2 & c \end{bmatrix}$$

$$A^T = A$$

$$w^T A w = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \beta & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$$

$$= \begin{bmatrix} \alpha x + \beta y & \beta x + d y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$$

$$N^2 \quad w \text{ dom } v$$

$$N+1$$

$$w = \begin{bmatrix} x & 1 & y \end{bmatrix}$$

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & bxy & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$\begin{matrix} ax & + & by & + & cy \\ \uparrow & & & & \uparrow \\ x & & & & y \end{matrix}$$

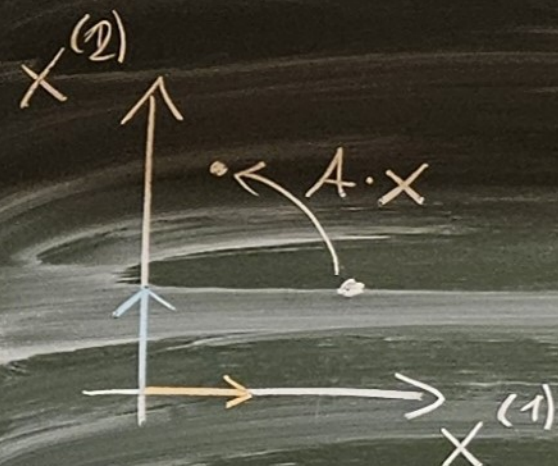
$$x_{n+1} = A x_n$$

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kat α



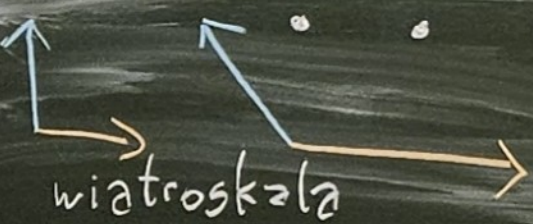
obrot

$$B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$



$$x = \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix}$$

$$A \cdot x$$

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix}$$

$$A^{-1} I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A \cdot ? = I \quad A^{-1} = \begin{bmatrix} 1/2 & 1/2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

1. obrot B

2. wiatroskala A

$$\begin{bmatrix} -5 \\ 1 \end{bmatrix}$$

1. wiatroskala A

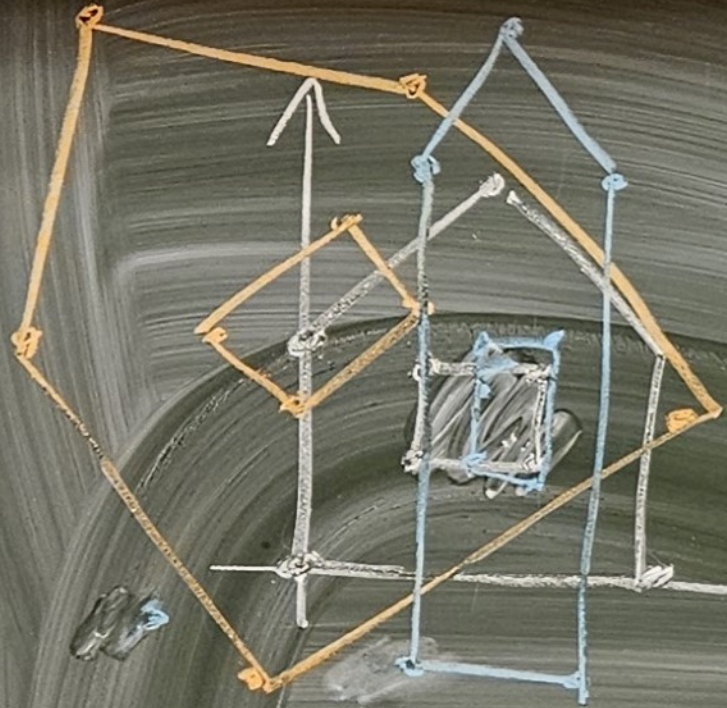
2. obrot B

$$A \cdot B = \begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 0 & -1 \\ 2 & -1 \end{bmatrix}$$

W dommu

$\det(A)$



A

B

$\det(B)$

$$\begin{bmatrix} x_1^{(1)} & x_2^{(1)} & x_3^{(1)} \\ x_1^{(2)} & x_2^{(2)} & x_3^{(2)} \end{bmatrix}$$