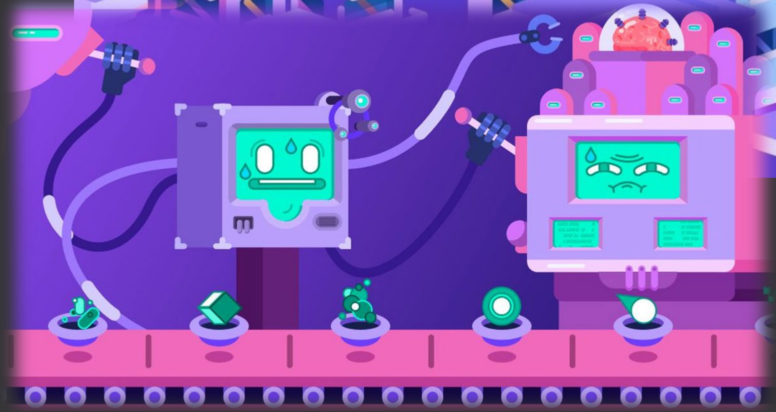


Metody i narzędzia *Big Data*

Obliczenia w małej i dużej skali - elementarzyk

Średnia i wariancja na bieżąco – *przyrostowo, dla całego sygnału*

Algorytmy do liczenia średniej i wariancji „w locie” mają ogólną postać

$$m_N = \Psi\left(m_{N-1}, x_N\right)$$

$$v_N = \Phi\left(v_{N-1}, m_N, x_N\right)$$

A konkretnie

$$m_N = \frac{N-1}{N} m_{N-1} + \frac{1}{N} x_N$$

$$v_N = \frac{N-1}{N} v_{N-1} + \frac{1}{N-1} \left(m_N - x_N\right)^2$$

Przyrostowo vs okno przesuwne

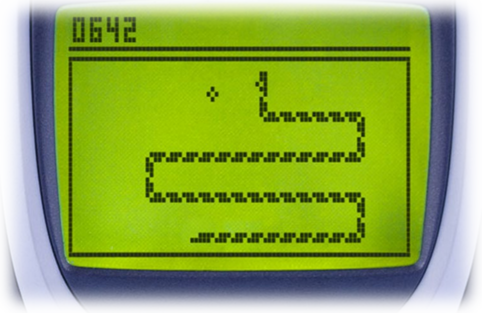
Przyrostowo

m_N

m_{N+1}

m_{N+2}

$\vdots$



Okno przesuwne

$m_{1,h}$

$m_{2,h+1}$

$m_{3,h+2}$

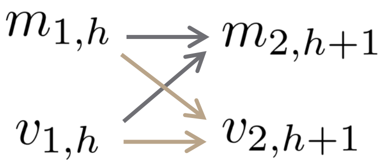
$\ddots$

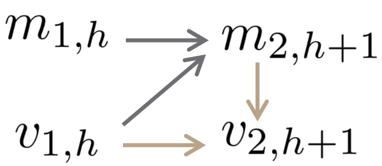
Średnia i wariancja na bieżąco – *na oknie przesuwnym*

Algorytmy do liczenia średniej i wariancji „w locie” mają ogólną postać

$$m_{2,h+1} = \Psi\left(m_{1,h}, x_{h+1}, x_1\right)$$

$$v_{2,h+1} = \Phi\left(v_{1,h}, m_{1,h}, x_{h+1}, x_1\right) \quad \text{lub} \quad v_{2,h+1} = \Phi\left(v_{1,h}, m_{2,h+1}, x_{h+1}, x_1\right)$$

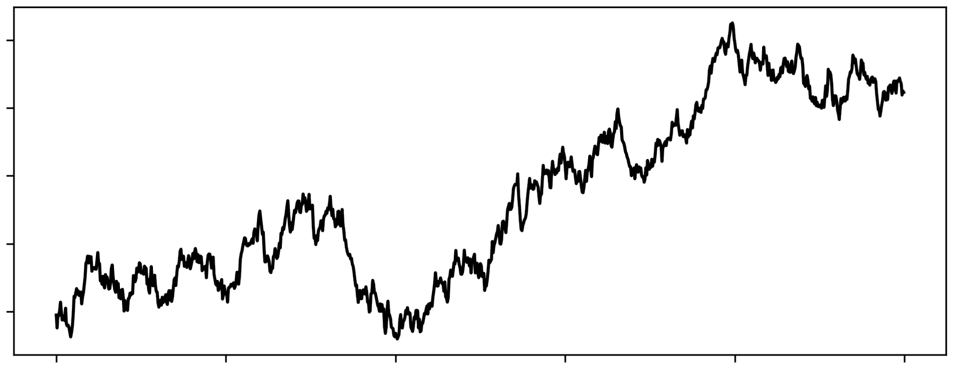
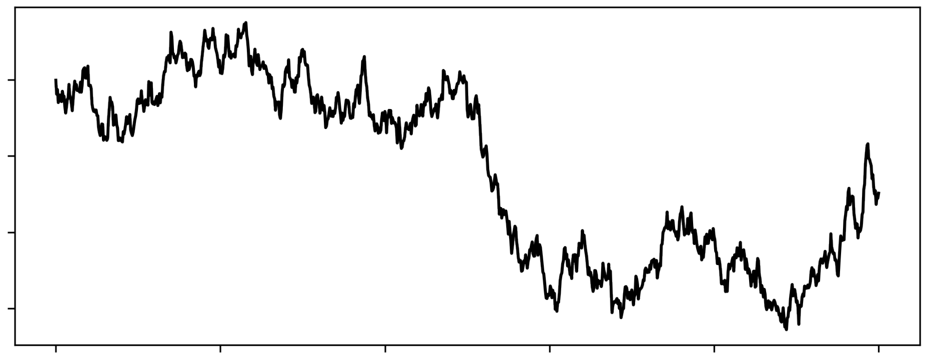
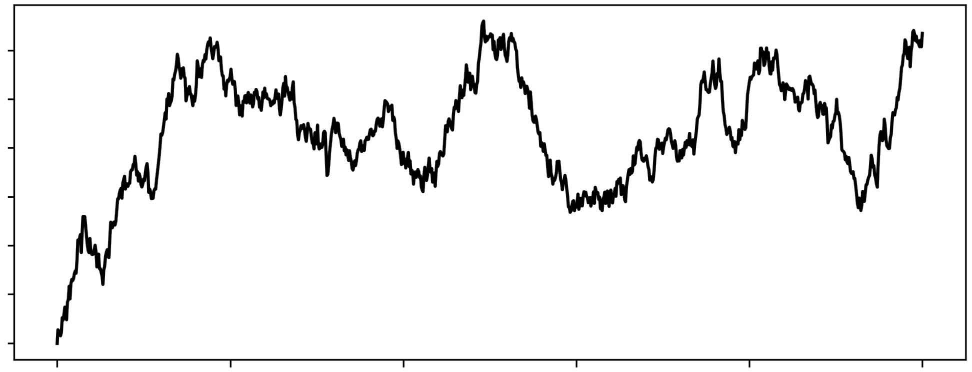
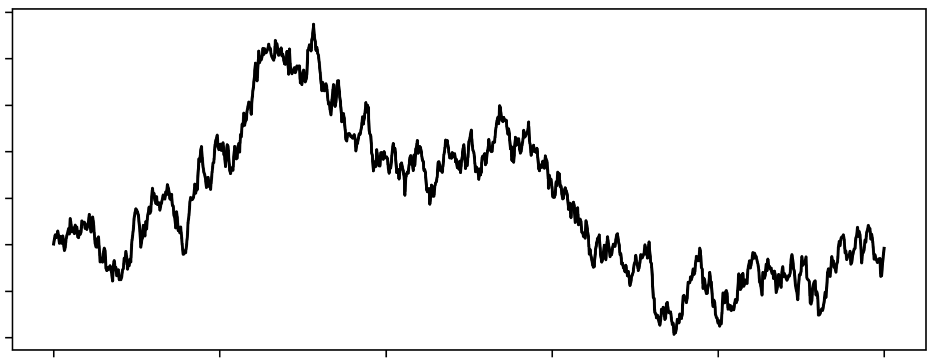




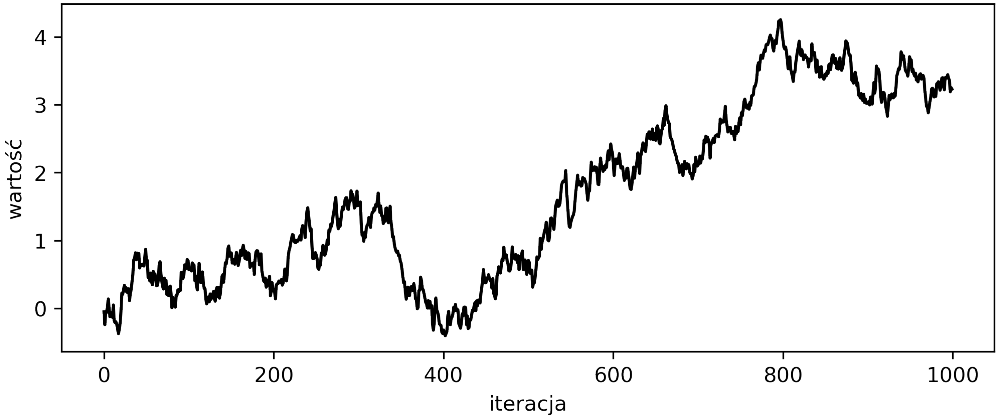
A konkretnie

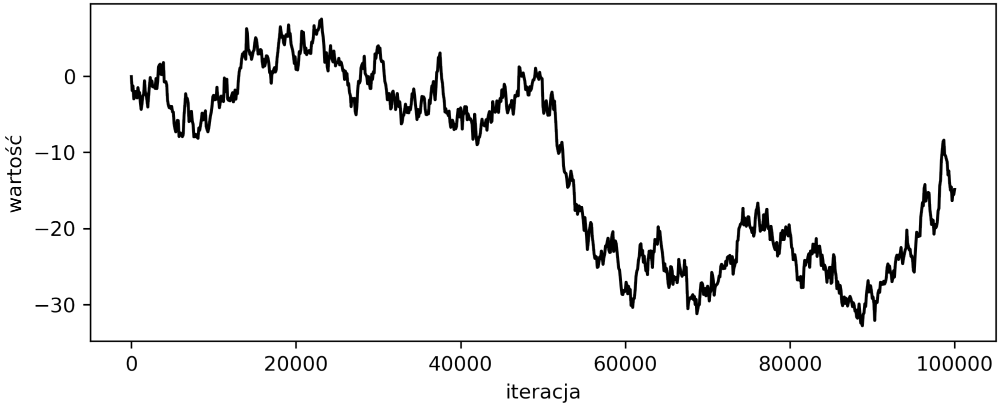
$$m_{2,h+1} = m_{1,h} + \frac{1}{h}\left(x_{h+1} - x_1\right)$$

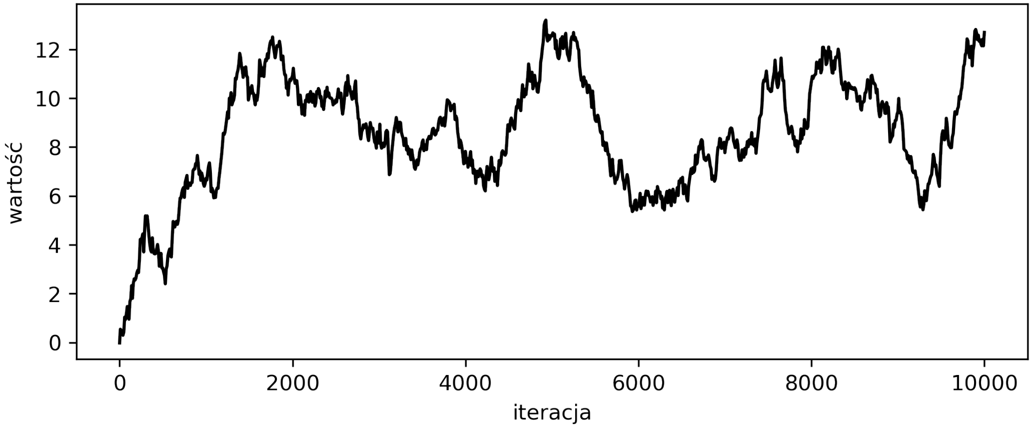
$$v_{2,h+1} = v_{1,h} - \frac{1}{h-1}\left(m_{1,h} - x_1\right)^2 + \frac{1}{h-1}\left(m_{1,h} - \frac{1}{h}x_1 + \frac{1-h}{h}x_{h+1}\right)^2$$

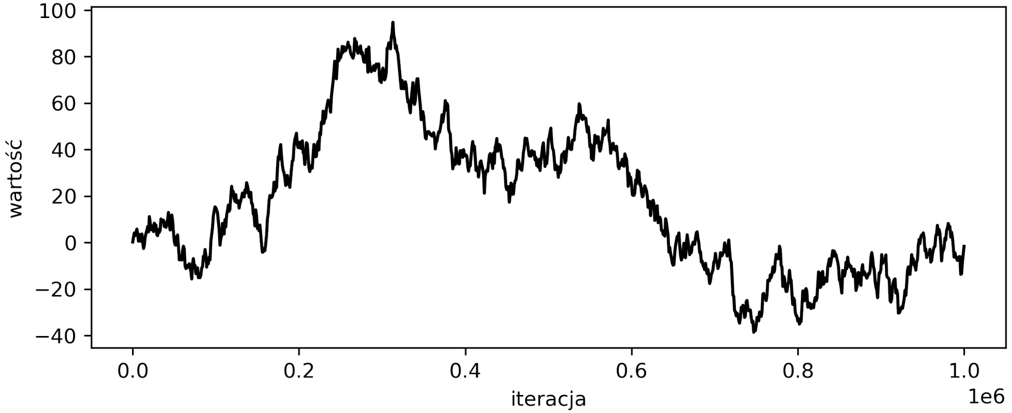





Skala procesu









Wykładnik Hursta

$$\{x_i\}_{i=1}^{N}$$

$$X_N = \sum_{i=1}^{N} x_i$$

$$R/S(N) = \frac{\max\limits_{1\leq i\leq N}\left\{X_i - \frac{i}{N}X_N\right\} - \min\limits_{1\leq i\leq N}\left\{X_i - \frac{i}{N}X_N\right\}}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(x_i - \frac{1}{N}X_N\right)^2}}$$

$$R/S(N) \propto N^{H}$$

https://pypi.org/project/nolds/

https://github.com/Mottl/hurst

Stochastic Models That Separate Fractal Dimension and Hurst Effect, Tilmann Gneiting and Martin Schlather (2001)

Qian, B., & Rasheed, K. (2004). *Hurst exponent and financial market predictability*. Proceedings of the IASTED International Conference Cambridge, MA.

Graves, T., Gramacy, R., Watkins, N., & Franzke, C. (2017). *A brief history of long memory: Hurst, Mandelbrot and the road to ARFIMA*. Entropy, 19(9).

Wykładnik Hursta

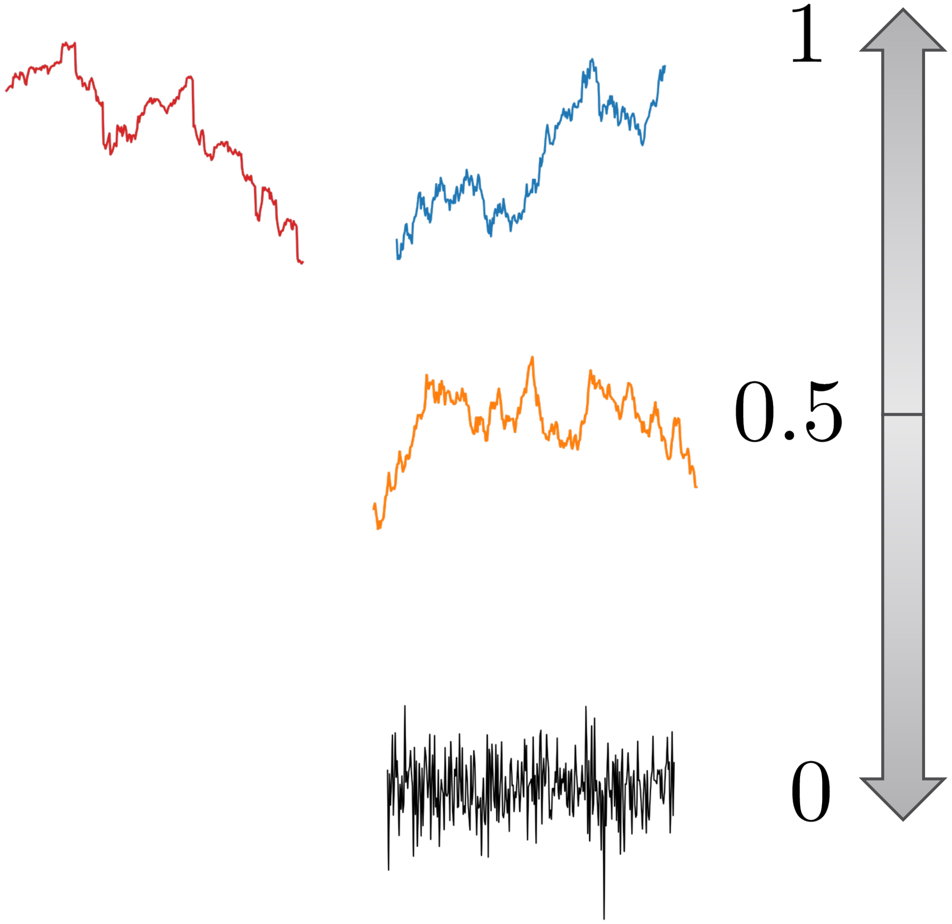
Inny sposób

$$y_i = \log x_i$$

$$V(\tau) \equiv \mathrm{Var}\left(\{ y_{i+\tau} - y_i \}_{i=1}^{N-\tau} \right)$$

$$V(\tau) \propto \tau^{2H}$$

Jak interpretować H ?



długa pamięć

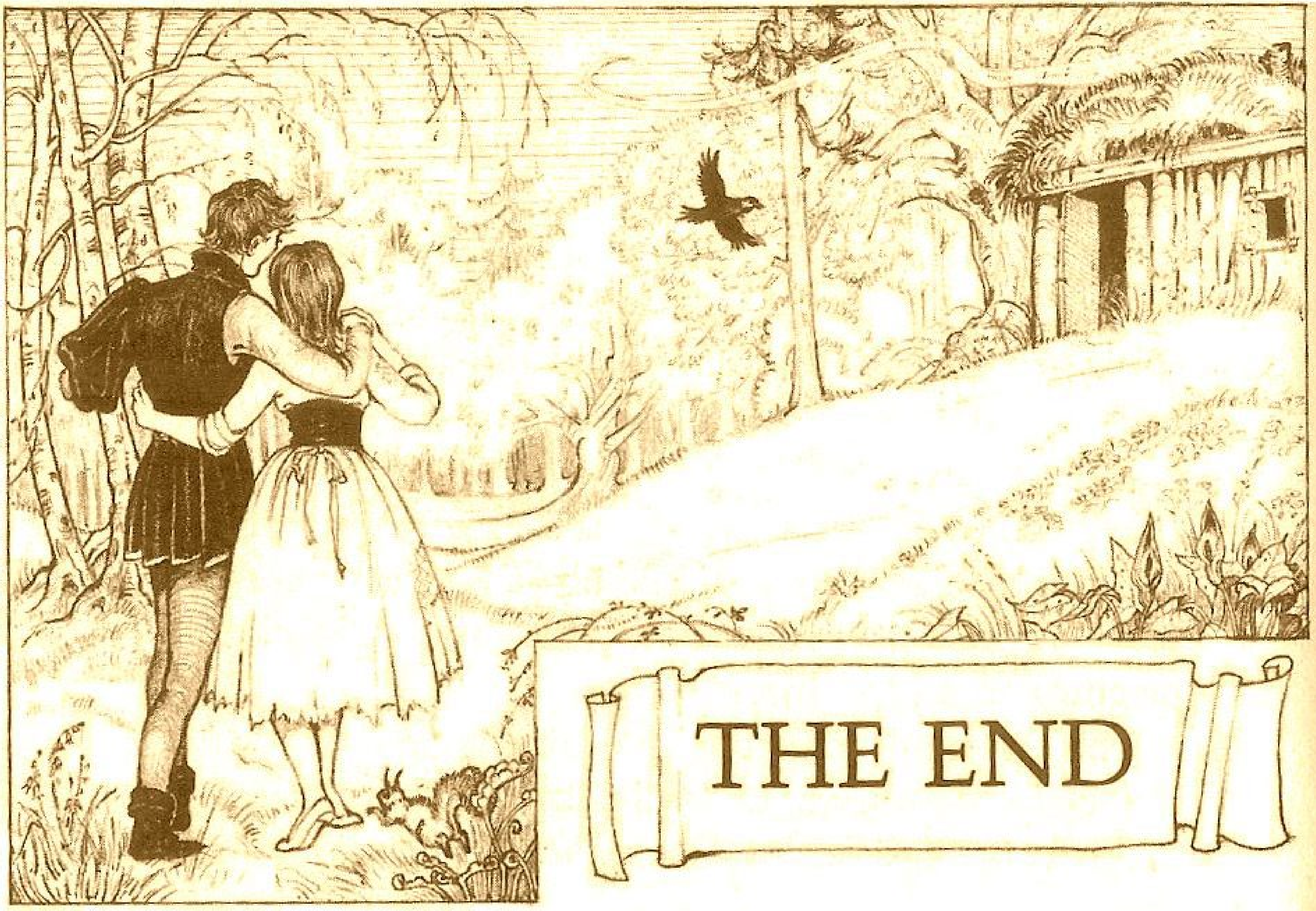
ang. *a trending* (*persistent*)

spacer losowy

krótka pamięć

ang. *mean-reverting* (*anti-persistent*)

długa pamięć


THE END